

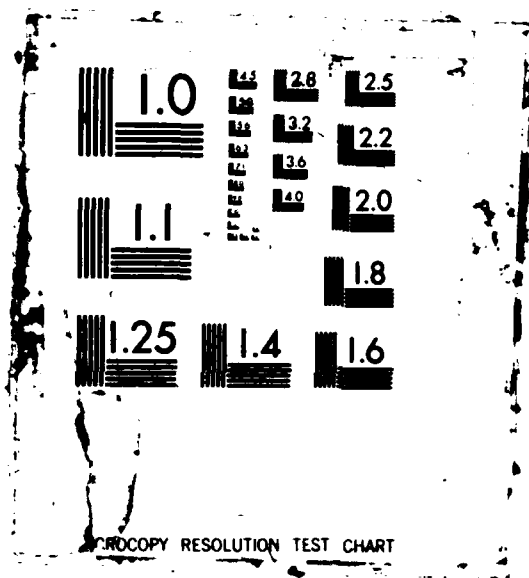
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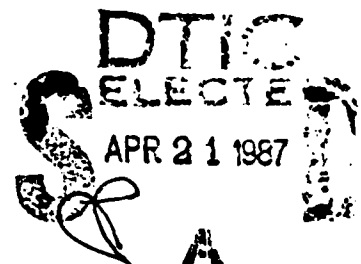
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In the second part a comprehensive study of the problem of synchronization over fading dispersive channels is presented. Synchronization is a fundamental problem in digital communications as used in mobile radio. In the first chapter, we consider a simple binary detection problem. The effect of receiver mismatch is investigated. A closed form expression for the probability of false alarm and probability of detection are given, also a set of curves are provided to demonstrate the amount of degradation for under-spread channels with some special scattering.

Chapter II deals with the performance of serial synchronizer over fading dispersive channels. The performance indices are the mean and variance of synchronization time. Upper and lower bounds on the mean and variance of synchronization time for a very general serial search system are derived and evaluated. It has been found that the results are highly dependent on the scattering function of the channel, the degree of spread, number of the cells used in the search procedure, and the time required to reject an incorrect cell when a false alarm occurs. An optimum closed loop structure for symbol synchronizer is derived in chapter III. The optimum synchronizer is similar to those already known symbol synchronizers that are being used over additive white Gaussian noise (AWGN) channels, except that the control signal is completely random; even in the absence of the AWGN. It is shown that the synchronizer structure is highly dependent on the scattering function of the channel. For a Gaussian shaped scattering function, a simple closed loop structure is obtained and expressions for tracking error statistics are derived.

In chapter IV the method of stochastic approximation is applied to the problem of synchronization over fading dispersive channels. A recursive estimation procedure is developed to estimate the two parameters needed by the synchronizer. The result is a closed loop in which the component of the error signal is proportional to the expected value of the derivative of the likelihood function with respect to the appropriate parameter.

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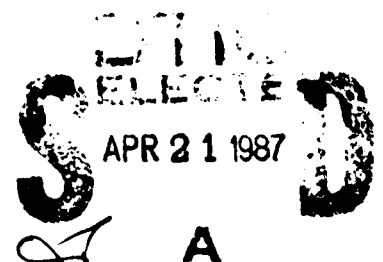
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SUMMARY

This report is made up of two parts. In the first part we present additional results on the performance of random access protocols for mobile packet radio networks. The effect of three different types of diversity reception in Nakagami fading environment on the channel throughput and the average packet delay of nonpersistent carrier sense multiple access (NPCSMA) protocol is considered. Expressions for the probability of packet error with diversity receptions are newly obtained for both independent and correlated diversity branches. A noncoherent frequency shift keying modulation scheme is assumed.

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PART I

PERFORMANCE EVALUATION OF DIVERSITY IN RANDOM ACCESS PROTOCOLS

I. Introduction

The effect of Rayleigh fading on the throughput of different random access protocols such as Aloha and NPCSMA have been introduced [1,2]. However, the Nakagami distribution [3] covers a wider class of fading than any other model. Experimental as well as theoretical [4,5] studies showed that it is the best fit for data obtained from a model of urban radio multipath channels. The effect of diversity reception on the probability of bit error with Nakagami fading was first considered in [6]. The analysis was confined to CPSK which is not suitable for mobile communications, beside that these results cannot apply to packet switching where we have packets of N bits. Some other results were obtained [7,8] for NCFSK with MRC and selection combining diversity techniques. Again these results were obtained for the probability of bit error as well as the third combining technique, namely equal gain combining, was not considered. The effect of Nakagami fading on the NPCSMA throughput was first considered in [9] for successive correlated packet retransmissions. However, the analysis of the diversity reception on the probability of packet error and consequently the channel throughput represents an uninvestigated case. In section II, the probability of packet error with MRC receiver is obtained when the diversity branches are independent with either identical parameters on each branch or not. The effect of correlation among two diversity branches is also considered.

Similar results are presented in Section III for selection combining. Section IV includes the results for equal gain combining. Modified expressions for the throughput and the delay are obtained in Section V for NPCSMA protocol along with conclusions.

II. Probability of block error of MRC diversity system:

The following assumptions are made to obtain the results:

- a) NCFSK modulation mode is considered.
- b) The fading on each branch is nonselective flat type of fading that is modeled using Nakagami distribution.
- c) The noise is additive white Gaussian with spectral height $2N_0$, and is the same at all branches. One of the known diversity techniques [10] is the MRC which allows the signal to noise ratios to be added noncoherently along the M branches. In other words

$$\gamma = \sum_{k=1}^M \gamma_k \quad (1)$$

The fading envelope $S(t)$ at the i^{th} branch has a Nakagami- m distribution as follows:

$$f(S) = \frac{2}{\Gamma(m_i)} \left(\frac{m_i}{\Omega_i}\right)^{m_i} S^{2m_i-1} \exp\left(-\frac{m_i}{\Omega_i} S\right) \quad (2)$$

where $\Omega_i = \langle S_i^2 \rangle$

$$m_i = \frac{\Omega_i^2}{\langle (S_i^2 - \Omega_i)^2 \rangle}, m_i \geq \frac{1}{2} \quad (3)$$

The signal to noise ratio at the i^{th} branch will be

$$\gamma_i = \frac{S_i^2}{2N_o} \quad (4)$$

The probability density function (p.d.f.) for γ_i will be

$$f(\gamma_i) = \frac{(k_i)^{m_i}}{\Gamma(m_i)} \gamma_i^{m_i-1} \exp(-k_i \gamma_i) \quad (5)$$

where

$$k_i = \frac{2N_o m_i}{\Omega_i} = \frac{m_i}{\gamma_i}, \bar{\gamma}_i = \text{Average SNR} \quad (6)$$

This can be recognized as a Gamma distribution. The probability of packet error is defined as follows

$$\bar{P}_{pe} = \int_0^{\infty} [1 - (1 - P_b)^N] f(\gamma) d\gamma \quad (7)$$

where N is the number of bits in each packet. P_b is the probability of bit error for a certain modulation scheme. For NCFSK P_b is given [12] by:

$$P_b = \frac{1}{2} e^{-\gamma/2}, \gamma \geq 0 \quad (8)$$

Case (A) : Independent Diversity Branches

For identical parameters m, Ω on all branches, we have

$$\bar{P}_{pe} = 1 - \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n \int_0^{\infty} e^{-n\gamma/2} f(\gamma) d\gamma \quad (9)$$

The sum of i.i.d. Gamma distribution with parameters m, k is also a Gamma distribution [13]. i.e.

$$f(\gamma) = \frac{(k)^{m_i}}{\Gamma(m_i)} \gamma^{m_i-1} \exp(-k\gamma) \quad (10)$$

where

$$m_i = Mm \quad (11)$$

Substituting for (10) in (9) and performing the integration we get

$$\bar{P}_{pe} = 1 - \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n \frac{1}{\left(1 + \frac{n}{2k}\right)^{mM}} \quad (12)$$

Equation (12) can be easily evaluated when N is not large. This is a reasonable assumption [14], since a large N would cause an unacceptable probability of error. If the parameters differ in each branch we have

$$\bar{P}_{pe} = 1 - \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n \text{L.T.}[f(\gamma)]_{S = \frac{n}{2}} \quad (13)$$

where L.T. is the Laplace transform operator. The characteristic (c/c) function for γ_i is given by [13]

$$= \left(\frac{K_i}{K_i - t}\right)^{m_i} \quad (14)$$

Then the c/c function for γ will take the form:

$$= \prod_{i=1}^M \left(\frac{K_i}{K_i - t}\right)^{m_i} \quad (15)$$

Consequently, the L.T. for $f(\gamma)$ will be

$$F(S) = \prod_{i=1}^M \left(\frac{K_i}{K_i + S}\right)^{m_i} \quad (16)$$

Substituting for (16) in (13) one gets:

$$\bar{P}_{pe} = 1 - \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n \prod_{i=1}^M \left(\frac{K_i}{K_i + \frac{n}{2}}\right)^{m_i} \quad (17)$$

For arbitrary branch parameters m_k, Ω_k but equal $\left(\frac{m_k}{\Omega_k}\right)$ for all branches we get

$$f_\gamma(\gamma) \sim \text{Gamma}(k, \sum_{k=1}^M m_k) \quad (18)$$

And consequently

$$\bar{P}_{pe} = 1 - \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n \prod_{i=1}^M \left(\frac{k}{k + n/2}\right)^{\sum_{k=1}^M m_k} \quad (19)$$

Case B: Correlated Branches ($M = 2$)

In this case we assume that the fading envelope has the same parameters in both branches. The envelope in this case will take the form [3]:

$$f(S) = \frac{2\sqrt{\pi}}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^{2m} \frac{S}{(1-\rho)^m} \exp\left[\frac{-mS^2}{\Omega(1-\rho)}\right] \cdot \left[\frac{\Omega(1-\rho)S^2}{2m\sqrt{\rho}}\right]^{m-\frac{1}{2}} I_{m-\frac{1}{2}}\left(\frac{m\sqrt{\rho}R^2}{\Omega(1-\rho)}\right) \quad (20)$$

where $I_X(\cdot)$ is the modified Bessel function of the first kind of order X . Substituting for equation (4) in (20) one gets:

$$f(\gamma) = \frac{2\sqrt{\pi} N_o}{\Gamma(m)(1-\rho)^m} \left(\frac{m}{\Omega}\right)^{2m} \exp\left(-\frac{2N_o m \gamma}{\Omega(1-\rho)}\right) \cdot \left(\frac{\Omega(1-\rho)(2N_o \gamma)}{2m\sqrt{\rho}}\right)^{m-\frac{1}{2}} \cdot I_{m-\frac{1}{2}}\left(\frac{2mN_o\sqrt{\rho}\gamma}{\Omega(1-\rho)}\right) \quad (21)$$

Hence the probability of packet error will take the form:

$$\bar{P}_{pe} = 1 - C \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n \cdot \int_0^{\infty} \exp\left(-\frac{n\gamma}{2}\right) \exp\left(-\frac{2N_o C_1 \gamma}{\sqrt{\rho}}\right) \cdot \left(\frac{2N_o \gamma}{2C_1}\right)^{m-\frac{1}{2}} I_{m-\frac{1}{2}}(2N_o C_1 \gamma) d\gamma \quad (22)$$

where

$$C = \frac{2N_o\sqrt{\pi}}{\Gamma(m)(1-\rho)^m} \left(\frac{m}{\Omega}\right)^{2m} \quad (23)$$

$$, C_1 = \frac{m\sqrt{\rho}}{(1-\rho)\Omega} \quad (24)$$

Rearranging (22), one obtains:

$$\bar{P}_{pe} = 1 - K \left(\frac{N_o}{C_1}\right)^{m-\frac{1}{2}} \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n$$

$$\int_0^{\infty} (\gamma)^{y-1} I_x(b\gamma) \exp(-a\gamma) d\gamma \quad (25)$$

where $x = m - \frac{1}{2}$, $y = m + \frac{1}{2}$,

$$a = \frac{n}{2} + \frac{2N_0 C_1}{\sqrt{\rho}}, \quad b = 2N_0 C_1 \quad (26)$$

The integration in equation (25) is found to be [15]

$$\frac{\Gamma(2m) \cdot (2N_0 C_1)^{m - \frac{1}{2}}}{\Gamma(m + \frac{1}{2}) 2^{m - \frac{1}{2}} \cdot (\frac{n}{2} + \frac{2N_0 C_1}{\sqrt{\rho}})^{2m}} \quad (27)$$

where, ${}_2F_1(\cdot)$ is the hypergeometric function.

The conditions to be satisfied for the above integration to exist are

$$\left. \begin{array}{l} (1) \quad x + y > 0 \\ (2) \quad |a| > |b| \\ (3) \quad \text{Re } |a| > \text{Re } |b| \end{array} \right\} \quad (28)$$

It is clear that equation (28) is satisfied for every value of n and m . Using the relations from [16]

$${}_2F_1(a, b, c, z) = {}_2F_1(b, a, c, z) \quad (29)$$

$${}_2F_1(b, a, a, z) = \left(\frac{1}{1-z}\right)^b \quad (30)$$

$$\Gamma(2m) = \frac{1}{\sqrt{2\pi}} 2^{2m - \frac{1}{2}} \Gamma(m) \Gamma(m + \frac{1}{2}) \quad (31)$$

one gets for \bar{P}_{pe} :

$$\bar{P}_{pe} = 1 - \left(\frac{2N_0 m}{\Omega \sqrt{1-\rho}}\right)^{2m} \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n$$

$$\frac{1}{\left[\left(\frac{n}{2} + \frac{2N_0 C_1}{\sqrt{\rho}}\right)^2 - (2N_0 C_1)^2\right]^m} \quad (32)$$

III. Probability of packet error

for selection combining:

For two diversity branches we have:

Case A: Independent fading on the diversity branches.

The resulting signal to noise ratio can be described by:

$$\gamma = \text{Max} (\gamma_1, \gamma_2) \quad (33)$$

Then, for arbitrary parameters, we get:

$$f(\gamma) = f(\gamma_1) |_{\gamma_1 = \gamma} F_{\gamma}(\gamma_2) + f(\gamma_2) |_{\gamma_2 = \gamma} F_{\gamma}(\gamma_1) \quad (34)$$

Then

$$\begin{aligned} f(\gamma) &= K_1^{m_1} \frac{1}{\Gamma(m_1)} \gamma^{m_1-1} \exp(-K_1 \gamma) \\ &\cdot \int_0^{\gamma} \frac{K_2^{m_2}}{\Gamma(m_2)} \gamma_2^{m_2-1} \exp(-K_2 \gamma_2) d\gamma_2 \\ &+ K_2^{m_2} \frac{1}{\Gamma(m_2)} \gamma^{m_2-1} \exp(-K_2 \gamma) \\ &\cdot \int_0^{\gamma} \frac{K_1^{m_1}}{\Gamma(m_1)} \gamma_1^{m_1-1} \exp(-K_1 \gamma_1) d\gamma_1 \end{aligned} \quad (35)$$

Consequently, the probability of packet error will be:

$$\begin{aligned} \bar{P}_{pe} &= 1 - \frac{K_1^{m_1} K_2^{m_2}}{\Gamma(m_1) \Gamma(m_2)} \left[\sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n \right. \\ &\quad \left. \left\{ \int_0^{\infty} \int_0^{\gamma} \exp\left(-\frac{n\gamma}{2} - K_1 \gamma\right) \right. \right. \\ &\quad \cdot \gamma^{m_1-1} \exp(-K_2 \gamma_2) \gamma_2^{m_2-1} d\gamma_2 d\gamma \\ &\quad \left. + \int_0^{\infty} \int_0^{\gamma} \exp\left(-\frac{n\gamma}{2} - K_2 \gamma\right) \gamma^{m_2-1} \right. \\ &\quad \left. \cdot \exp(-K_1 \gamma_1) \gamma_1^{m_1-1} d\gamma_1 d\gamma \right\} \end{aligned} \quad (36)$$

In the above integrals let $x = \gamma_i/\gamma$, $i=1,2$, Then we get:

$$\begin{aligned} \bar{P}_{pe} &= 1 - \frac{K_1^{m_1} K_2^{m_2}}{\Gamma(m_1) \Gamma(m_2)} \left[\sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n \right. \\ &\quad \left. \left\{ \int_0^1 x^{m_1-1} \int_0^{\infty} \exp\left(-\gamma\left(\frac{n}{2} + K_1 + K_2 x\right)\right) \right. \right. \end{aligned}$$

$$\cdot \gamma^{m_1 + m_2 - 1} d\gamma dx + \int_0^1 x^{m_1 - 1} \int_0^\infty \cdot \exp - \gamma \left(\frac{n}{2} + K_2 + K_1 x \right) \gamma^{m_1 + m_2 - 1} d\gamma dx \} \quad (37)$$

Or using the definition of the hypergeometric function and making some manipulations, we arrive at:

$$\begin{aligned} \bar{P}_{pe} = & 1 - \frac{K_1^{m_1} K_2^{m_2}}{B(m_1, m_2)} \sum_{n=0}^N \left(-\frac{1}{2}\right)^n \binom{N}{n} \\ & \cdot \left\{ \frac{1}{m_1} \cdot \frac{1}{\left(\frac{n}{2} + K_2\right)^{m_1 + m_2}} \right. \\ & \cdot {}_2F_1\left(m_1 + m_2, m_1, m_1 + 1, \frac{-K_1}{K_2 + \frac{n}{2}}\right) \\ & + \frac{1}{m_2} \frac{1}{\left(\frac{n}{2} + K_1\right)^{m_1 + m_2}} \\ & \cdot {}_2F_1\left(m_1 + m_2, m_2, m_2 + 1, \frac{-K_2}{K_1 + \frac{n}{2}}\right) \} \end{aligned} \quad (38)$$

Case (B): Correlated Branches:

The joint pdf of two SNR's γ_1, γ_2 whose envelopes are jointly random variables is given by [9]:

$$\begin{aligned} f(\gamma_1, \gamma_2) = & K' (\gamma_1 \gamma_2)^{\frac{m-1}{2}} \\ & \cdot \exp - \frac{K}{1-\rho} (\gamma_1 + \gamma_2) I_{m-1}(K'' \sqrt{\gamma_1 \gamma_2}) \end{aligned} \quad (39)$$

where

$$K' = \frac{K^{m+1}}{\Gamma(m)(1-\rho)(\rho)^{\frac{m-1}{2}}}, K'' = \frac{2\sqrt{\rho}k}{1-\rho} \quad (40)$$

Because of the symmetry of γ_1, γ_2 in (39), one can write for $f(\gamma)$:

$$f(\gamma) = 2 \int_0^\gamma f(\gamma, \gamma_1) d\gamma_1 \quad (41)$$

Hence

$$\bar{P}_{pe} = 1 - 2 \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n$$

$$\int_0^\infty \int_0^\gamma \exp(-\frac{n\gamma}{2}) f(\gamma, \gamma_1) d\gamma_1 d\gamma \quad (42)$$

Let $\gamma x = \gamma_1$ in (42) and change the order of integrations we obtain using [15]:

$$\begin{aligned} \bar{P}_{pe} &= 1 - 2K' \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n \\ &\cdot \int_0^1 \frac{x^{\frac{m-1}{2}} \Gamma(2m) (K''\sqrt{x})^{m-1}}{2^{m-1} \Gamma(m) \left(\frac{n}{2} + \frac{K}{1-\rho} (1+x)\right)^{2m}} \\ &\cdot {}_2F_1\left(m, m + \frac{1}{2}, m, \left(\frac{K''\sqrt{x}}{\frac{n}{2} + \frac{K}{1-\rho} (1+x)}\right)^2\right) dx \end{aligned} \quad (43)$$

It is obvious that the conditions for the integration to exist are satisfied

$$\forall 0 \leq x \leq 1, 0 \leq |\rho| < 1.$$

Using a similar approach to case A and after some simple manipulations one arrives at:

$$\begin{aligned} \bar{P}_{pe} &= 1 - L \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n \\ &\int_0^1 \frac{x^{m-1} [ax + b]}{[(ax + b)^2 - 4\rho a^2 x]^{m + \frac{1}{2}}} dx \end{aligned} \quad (44)$$

where

$$L = 2 \frac{\Gamma(2m)}{\Gamma(m)\Gamma(m)} \left(\frac{k^2}{1-\rho}\right)^m \quad (45)$$

$$, a = \frac{K}{1-\rho} \quad (46)$$

The above integration can be easily evaluated $\forall n$ numerically.

IV. Probability of Packet (block) Error for Equal Gain Combining

For M diversity branches with identical parameters, the fading envelope can be described by

$$S(t) = \sum_{k=1}^M S_k(t) \quad (47)$$

Case A Independent diversity branches:

From [3], the p.d.f. of $S(t)$ can be approximated by:

$$f(S) = \frac{2}{\Gamma(m_T)} \left(\frac{m_T}{\Omega_T}\right)^{m_T} S^{2m_T-1} \exp - \frac{m_T}{\Omega_T} S^2 \quad (48)$$

where

$$\left. \begin{aligned} m_T &= Mm \\ \Gamma_T &= M\Omega + \frac{M(M-1)\Omega}{m} \left\{ \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)} \right\}^2 \\ &= M^2\Omega(1 - \frac{0.2}{m}) \end{aligned} \right\} \quad (49)$$

The probability of packet error will take the form:

$$\begin{aligned} \bar{P}_{pe} &= 1 - \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n \\ &\int_0^{\infty} S^{2m_T-1} \frac{2}{\Gamma(m_T)} \left(\frac{m_T}{\Omega_T}\right)^{m_T} \\ &\cdot \exp - (K_T + \frac{n}{2}) S^2 dS \end{aligned} \quad (50)$$

Let $\frac{S^2}{2N_T} = \gamma$ and integrate, we get

$$\begin{aligned} \bar{P}_{pe} &= 1 - \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n \\ &\left(\frac{K_T}{K_T + n/2}\right)^{m_T}, K_T = \frac{2N_T m_T}{\Omega_T} \end{aligned} \quad (51)$$

$$, N_T = MN_o \quad (52)$$

Case B: Correlated diversity branches (M = 2)

We assume that the two branches have identical parameters (m,Ω). The p.d.f. of the combined output will be

$$f(S) = \int_0^S f(S_1, S-S_1) dS_1 \quad (53)$$

Hence,

$$\bar{P}_{pe} = 1 - \frac{4}{\Gamma(m)(1-\rho) \rho(\frac{m-1}{2})} \left(\frac{m}{\Omega}\right)^{m+1} \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n$$

$$\begin{aligned}
 & \cdot \int_0^{\infty} \exp\left(-\frac{nR^2}{2N_T}\right) \int_0^R [R_1(R-R_1)]^m \exp\left(-\frac{m}{\Omega(1-\rho)}\right) \\
 & \cdot [R_1^2 + (R-R_1)^2] \\
 & \cdot I_{m-1}\left(\frac{2m\sqrt{\rho}R_1(R-R_1)}{(1-\rho)\Omega}\right) dR_1 dR
 \end{aligned} \tag{54}$$

If we let $R_1 = Rx$, we get

$$\begin{aligned}
 \bar{P}_{pe} &= 1 - \frac{L}{2} \sum_{n=0}^N \left(-\frac{1}{2}\right)^n \binom{N}{n} \int_0^1 x^m (1-x)^m \\
 & \cdot \int_0^{\infty} \exp\left[-R^2\left(\alpha(x^2 + (1-x)^2) + \frac{n}{2}\right)\right] \\
 & \cdot R^{2m} I_{m-1}\left(\frac{2m\sqrt{\rho}}{(1-\rho)\Omega} R^2 x(1-x)\right) dR dx
 \end{aligned} \tag{55}$$

where L and α are constants,

$$L = \frac{4}{\Gamma(m)(1-\rho) \rho^{\frac{m-1}{2}}} \left(\frac{m}{\Omega}\right)^{m+1}, \alpha = \frac{m}{\Omega(1-\rho)} \tag{56}$$

Using [15], and after satisfying and regularity conditions, we get

$$\begin{aligned}
 \bar{P}_{pe} &= 1 - \sum_{n=0}^N \binom{N}{n} \left(-\frac{1}{2}\right)^n L_1 \\
 & \int_0^1 \frac{[x(1-x)]^{2m-1} P(n,x)}{[P^2(n,x) - P_1^2(x)]^{m+\frac{1}{2}}} dx
 \end{aligned} \tag{57}$$

where

$$P(n,x) = \frac{n}{2} + \frac{K}{1-\rho} [x^2 + (1-x)^2] \tag{58}$$

$$P_1(x) = \frac{2K\sqrt{\rho}}{1-\rho} x(1-x) \tag{59}$$

$$L_1 = 2\left(\frac{K^2}{1-\rho}\right)^m \frac{\Gamma(2m)}{\Gamma^2(m)} \tag{60}$$

V. Modified Expressions for the Channel Throughput for NPCSMA Scheme

The channel throughput and the average packet delay for NPCSMA protocol are given by [17]

$$\left. \begin{aligned} S &= \frac{G \exp(-aG)}{G(1 + 2a) + \exp(-aG)} \\ D &\cong \alpha + 1 + 2a + \left(\frac{G}{S} - 1\right)(1 + 2a + \delta + \alpha) \end{aligned} \right\} \quad (61)$$

The modified expressions for S and D , if the packet and acknowledgement channels are fading and identical, will be

$$\left. \begin{aligned} S_m &= S(1 - \bar{P}_{pe})^2 \\ D_m &\cong \alpha + 1 + 2a + \left(\frac{G}{S_m} - 1\right)(1 + 2a + \delta + \alpha) \end{aligned} \right\} \quad (62)$$

For the last 3 diversity schemes, we can always compute the expressions for S_m and D_m to investigate the effect of different combining techniques on the channel throughput.

Numerical Results

The improvement in the throughput and the delay of a channel with Nakagami fading using MRC diversity technique is displayed in figures 1 through 8. It is obvious that the performance approaches that of a nonfading channel with 4 diversity branches. However the approach is quicker when the branches have different parameters. Correlation among two diversity branches is shown to greatly affect the performance. Consequently, serious attention must be directed to assure independence between branches. It is also noticed that the limiting case where $\rho \rightarrow 1$ is achieved for MRC at a SNR 3 dB less than that needed for a single channel. This is expected from equation (1). Similar results are presented for both SGC and EGC. For SGC the performance degrades to reach that of a single channel at $\rho \rightarrow 1$. For SGC the limiting case as $\rho \rightarrow 1$ occurs at an average SNR 1.5 dB less than that needed for a single channel. It is also to be noticed that for SGC a high SNR branch along with the worst fading figure (m), the performance is better than that of two identical branches with relatively good fading figure. This implies that the average SNR affects the performance in a more effective way than the fading figure (which describes the severity of fading) does. The MRC technique shows a better performance followed by EGC and then the SGC. However, the improvement in performance of MRC over EGC is not attractive to favor MRC over EGC keeping in mind that EGC is easier to implement than MRC. The results obtained here contain the case of Rayleigh fading (at $m = 1$) as well as the case of single bit transmission (at $N = 1$).

VI. Conclusions:

The probability of packet error for three different diversity combining techniques is evaluated for independent fading envelopes over each branch. The effect of correlation among two different branches was also considered. The fading was modeled according to the Nakagami- m distribution. The results were used to obtain modified expressions for the channel throughput and the average packet delay for NPCSMA protocol.

References

- [1] J.A. Roberts and T.J. Healey, "Packet radio performance over slow Rayleigh fading channels," *IEEE Trans. on Comm.*, Vol. COM-28, pp. 279-286, Feb., 1980.
- [2] R. Sinha and S.C. Gupta, "Carrier sense multiple access for mobile packet radio channels: performance evaluation," *Proceedings of Conference on Information Sciences and Systems*, Princeton, N.J., March 1984.
- [3] M. Nakagami, "The m-distribution - A general formula of intensity distribution of rapid fading," *Statistical Methods in Radio Wave Propagation*, W.G. Hoffman (ed.), Pergamon Press, N.Y., London, pp. 3-36, 1960.
- [4] H. Suzuki, "A statistical model for urban radio propagation," *IEEE Trans. on Comm.*, Vol. COM-25, pp. 673-679, July 1977.
- [5] G.L. Turin, et al., "Simulation of urban vehicle monitoring systems," *IEEE Trans. on Veh. Tech.*, Vol. VT-21, Feb. 1972.
- [6] Y. Miyagaki, N. Morinaga, and T. Namekawa, "Error probability characteristics for CPSK signal through m-distributed fading channel," *IEEE Trans. on Comm.*, Vol. COM-26, pp. 88-99, Jan. 1978.
- [7] E.K. Al-Hussaini and A.M. Al-Bassiouni, "Performance of an ideal switched diversity receiver for NCFSK signals with Nakagami fading," *Trans. IECE Japan*, Vol. E65, pp. 750-751, Dec. 1982.
- [8] E.K. Al-Hussaini and A.M. Al-Bassiouni, "Performance of MRC diversity systems for the detection of signals with Nakagami fading," *IEEE Trans. on COM*, Vol. COM-33, pp. 1315-1319, Dec. 1985.
- [9] Hazem Tawfik and S.C. Gupta, "Performance Evaluation of NPSCMA protocol with Nakagami fading channel and correlated packet transmission," to be presented at the 1987 IEEE Phoenix Conference on Computers and Communications, Feb. 1987.
- [10] D.G. Brennan, "Linear Diversity Combining Techniques," *Proceed. of IRE*, Vol. 47, pp. 1075-1102, June 1959.
- [11] R.E. Eaves and A.H. Levesque, "Probability of block error for very slow Rayleigh fading in Gaussian noise," *IEEE Trans. on Comm.*, Vol. COM-25, pp. 368-373, March 1977.
- [12] S. Stein, W.R. Bennett and M. Schwartz, *Communication Systems and Techniques*, McGraw-Hill Book Co., Inc., N.Y. 1966.
- [13] Mood, Graybill and Boes, *Introduction to the Theory of Statistics*, McGraw-Hill series in probability and statistics, N.Y., 1974.
- [14] H. Tawfik and S.C. Gupta, "Optimal packet length for NPSCMA scheme in Mobile fading channels," to be presented at the 1987 IEEE Region 5 Conference, March, 1987.
- [15] W. Gröbner and N. Hofreiter, *Integraltafel - Bestimmte Integrale*, Springer-Verlag, Vienna, Austria, 1961.

- [16] A. Erdelyi et al., *Higher Transcendental Functions*, Vol. I, II, McGraw-Hill Book Corp., N.Y. 1953.
- [17] L. Klienrock and F.A. Tobagi, "Packet Switching in Radio Channels: Part I: Carrier Sense Multiple Access Modes and their Throughput-Delay Characteristics," *IEEE Trans. on Comm.*, Vol. COM-23, pp. 1400-1416, December 1975.

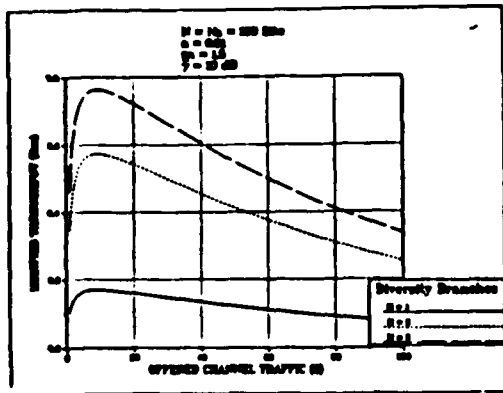


Fig. (1) Throughput versus traffic for NPCSMA protocol with MRC diversity reception (M independent identical branches) through Nakagami fading channel.

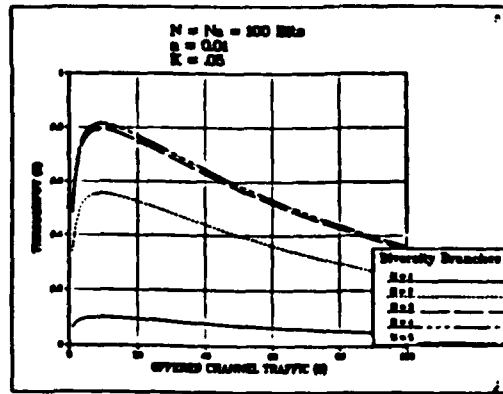


Fig. (3) Throughput versus traffic with MRC diversity reception (Different independent branches and equal K's).

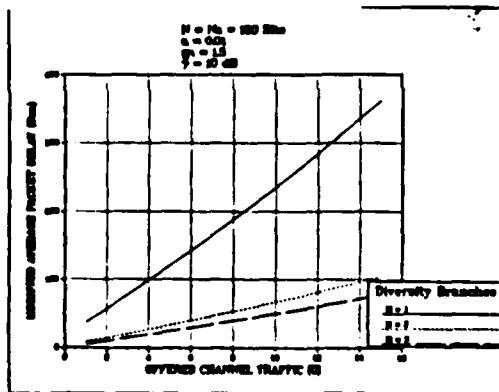


Fig. (2) Delay versus traffic for NPCSMA protocol with MRC diversity reception (M independent identical branches) through Nakagami fading channel.

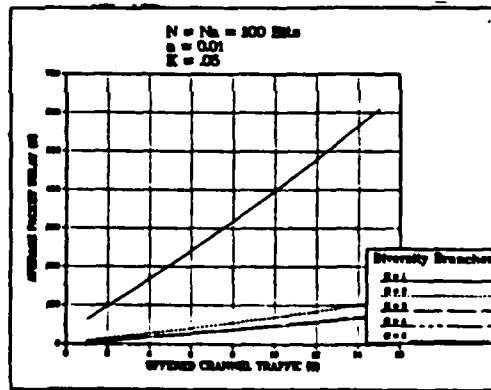


Fig. (4) Delay versus traffic with MRC diversity reception (Different independent branches and equal K's).

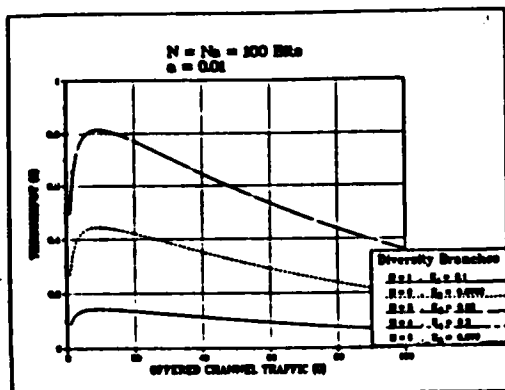


Fig. (5) Throughput versus traffic with MRC diversity reception (M independent distinct branches).

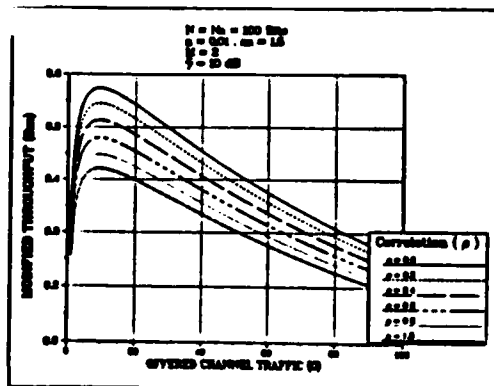


Fig. (7) Effect of correlation, among two diversity branches in MRC case with identical branch parameters, on the channel throughput.

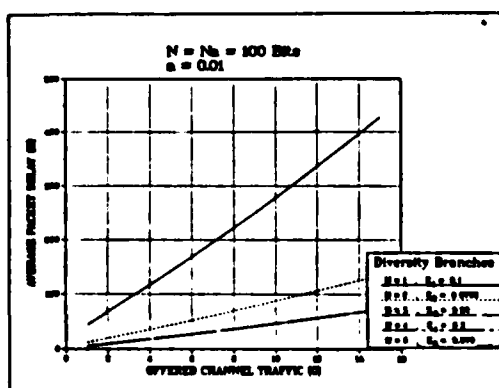


Fig. (6) Delay versus traffic with MRC diversity reception (M independent distinct branches).

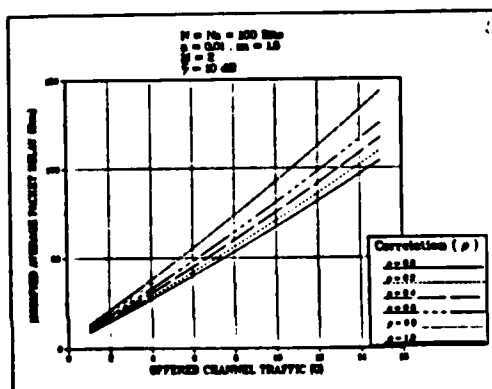


Fig. (8) Effect of correlation, among two diversity branches in MRC case with identical branch parameters, on the channel delay.

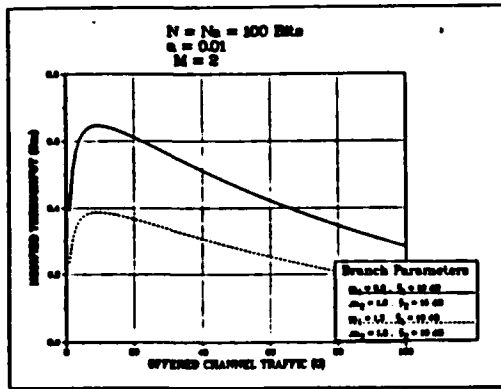


Fig. (9) Comparison of two SGC diversity systems with different independent branch parameters with respect to the channel throughput.

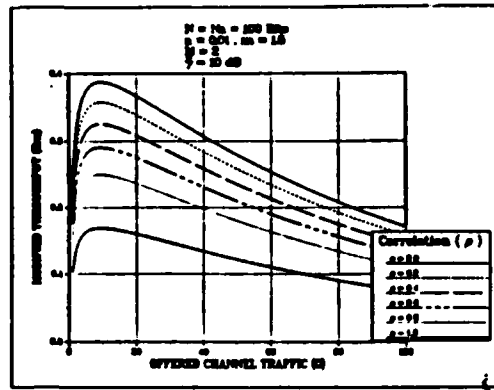


Fig. (11) Effect of correlation among diversity branches for SGC technique on the channel throughput.

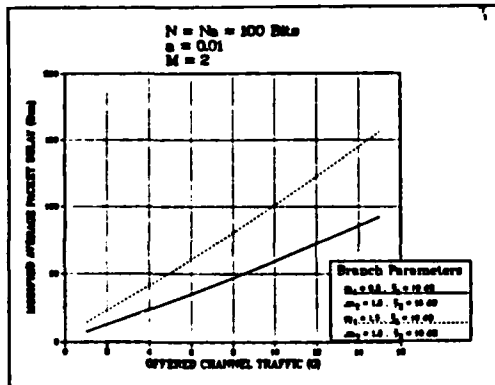


Fig. (10) Comparison of two SGC diversity systems with different independent branch parameters with respect to the average packet delay.

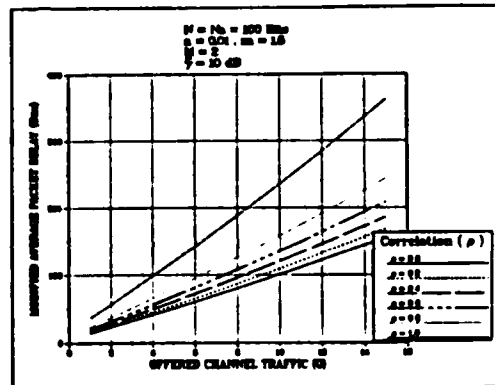


Fig. (12) Effect of correlation among diversity branches for SGC technique on the channel delay.

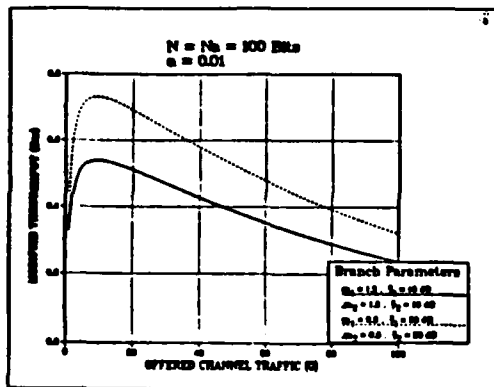


Fig. (13) Comparison of two diversity systems using EGC technique but with different parameters with respect to the channel throughput.

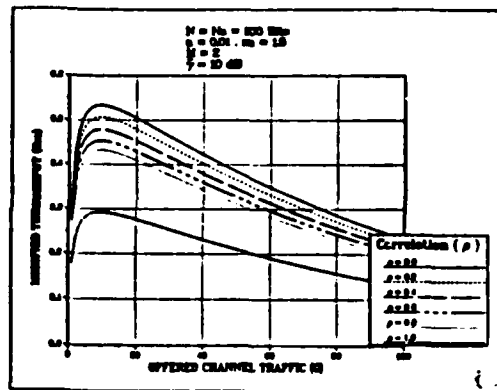


Fig. (15) Effect of correlation among diversity branches for EGC technique on the channel throughput.

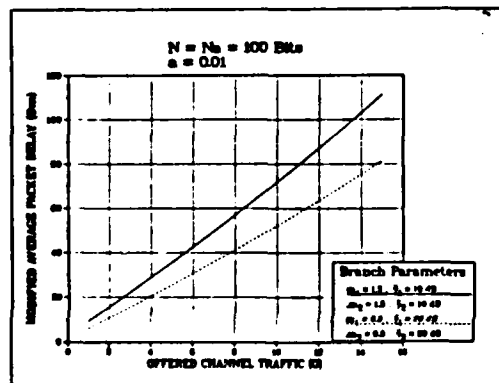


Fig. (14) Comparison of two diversity systems using EGC technique but with different parameters with respect to the average delay.

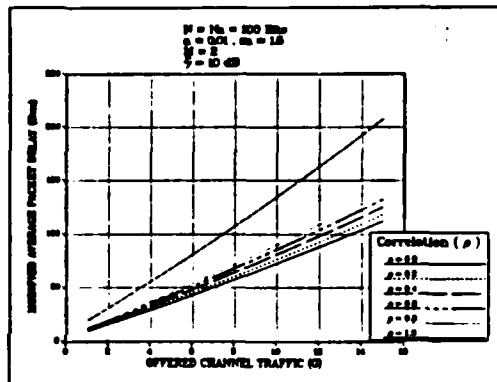


Fig. (16) Effect of correlation among diversity branches for EGC technique on the average packet delay.

PART II

SYNCHRONIZATION OVER FADING DISPERSIVE CHANNELS

CHAPTER I

PERFORMANCE DEGRADATION DUE TO RECEIVER MISMATCH IN COMMUNICATION OVER FADING DISPERSIVE CHANNELS †

The engineering importance of fading dispersive channels has increased markedly in the recent years. Optimum detection over such channels requires a perfect knowledge of the channels scattering functions which completely characterize the behavior of fading dispersive channels. Lack of such knowledge may result in performance degradation. The objective of this study is to analyze the performance degradation under mismatch conditions. Two types of receiver mismatch will be considered. First, the receiver is assumed to have knowledge of the shape of the scattering function but the mean time delay and the mean frequency shift are unknown. Secondly, we will investigate the performance degradation when the shape of the scattering function is not known to the receiver. A closed form expression for the probability of false alarm and probability of detection are given, also a set of curves are provided to demonstrate the amount of degradation for under-spread channels with some special scattering functions.

1.1 INTRODUCTION

During recent years there has been an increasing amount of attention given to the study of fading dispersive channels, channels that exhibit both fading and dispersion. The determination of optimum modulation and demodulation techniques and the evaluation of the efficacy of optimum and suboptimum receivers for such channels are of major importance to the system designer.

Fading dispersive channels are usually best described as random, linear, time-invariant filters [1]. The characterization of time-variant, linear filters in

terms of system functions received its first general analytical treatment by Zadah [2], who introduced the time-variant transfer function and the bi-frequency function, as frequency domain methods of characterizing time-variant linear filters. There is another common approach to describe the channel, an approach which involves the notion of scatterers. This approach leads to a physical picture of the channel as a continuum of moving scatterers. In such model, propagation is established by a single scattering from a large number of independent scatterers.

In many applications, it is reasonable to suppose that the impulse response of the filter is a sample function of a Gaussian random process. Using such supposition, the specification of the channel reduces to the specification of the mean and correlation functions either of the channel's random impulse response (first approach) or of the received process conditioned upon the transmitted waveform (second approach). Many channels are adequately modeled by taking the mean to be zero and the correlation function to be a special form, which is determined by the scattering function of the channel as will be seen in the next section.

In section 1.2 mathematical model of the channel is presented, and the statistics of the output of the channel are given in terms scattering function of the channel and the complex envelope of the transmitted waveform.

In section 1.3 we present the receiver's structure and discuss a way of obtaining an approximation to the probability of detection and the probability of false alarm. The discussion in this section is limited to special category of problems; namely, the low-energy-coherence problems.

In section 1.4 we investigate the degradation in performance due to the mismatched receiver, by that it is meant that the receiver has not enough

information regarding the scattering function of the channel. The two cases of interest are

- (1) The shape of the scattering is known, except for some unknown parameters.
- (2) The scattering function of the channel is completely unknown to the receiver.

1.2 CHANNEL MODEL

The channel to be considered is depicted in Figure 1.1. This channel is completely described by the scattering function $\sigma(r,f)$. The most important parameters of $\sigma(r,f)$ are B , the frequency interval in f outside of which $\sigma(r,f)$ is essentially zero, and L , the time interval in r outside of which $\sigma(r,f)$ is effectively zero. The quantity B is called the Doppler spread, and represents the average amount that an input waveform will be spread in frequency, while L is known as the multipath spread, and represents the average amount in time by which an input signal will be spread. The product $S=BL$ is called the spread of the channel, channels for which $S < 1$ are called under-spread channels, while channels with $S > 1$ are called over-spread channels. Characterization and classification of fading dispersive channels are given in [3].

Throughout the discussion, we denote the channel input and output waveforms by $s(t)$ and $y(t)$, respectively. These waveforms will usually be represented by their complex envelopes, thus

$$s(t)=\text{Re}\left\{u(t)\exp\left[j\omega_0 t\right]\right\}, \quad (1.1-a)$$

$$y(t)=\text{Re}\left\{v(t)\exp\left[j\omega_0 t\right]\right\}, \quad (1.1-b)$$

where $\text{Re} \{.\}$ denotes "the real part" of the indicated quantity, and ω_0 denotes the nominal carrier frequency in radians per second.

In many applications it is reasonable to assume that the received signal is conditionally Gaussian. By conditionally Gaussian, we mean that given the transmitted waveform, the received waveform is Gaussian random process. If one accepts this supposition, the description of the channel reduces to the specification of the mean and correlation function of the received process conditioned upon the transmitted waveform. It can be shown [3] that

$$R_y(t, \tau) = \text{Re} \left\{ R(t, \tau) \exp [j\omega_0(t - \tau)] \right\}, \quad (1.2)$$

where

$$R(t, \tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma(r, f) u(t - r) u^*(\tau - r) \exp [j2\pi f(t - \tau)] dr df. \quad (1.3)$$

The function $R(t, \tau)$ defined by (1.3) is the complex correlation function of the complex envelope of $y(t)$, that is

$$R(t, \tau) = E \left\{ v(t) v^*(\tau) \right\}. \quad (1.4)$$

Insight into the scattering function and methods of measuring it are discussed in [4]

1.3 RECEIVER STRUCTURE

Optimum receiver structure for digital signaling over fading dispersive channels have been determined [5], and the performance of binary signaling systems has been thoroughly investigated. In this section a brief discussion of the optimum receiver structure for simple binary detection problem is presented. The received waveforms under the two hypotheses are

$$\begin{aligned}
H_1: \quad z(t) &= v(t) + w(t), & T_1 \leq t \leq T_f, \\
H_0: \quad z(t) &= w(t), & T_1 \leq t \leq T_f,
\end{aligned} \tag{1.5}$$

We assume that that $w(t)$ is a complex white, zero-mean Gaussian process with spectral height N_0 and $v(t)$ is zero-mean complex Gaussian random process with covariance function $R(t, \tau)$.

The approach to design the optimum receiver is analogous to the approach used in the deterministic signal case [6]. It can be shown that the optimum receiver compares the likelihood ratio, l , with a threshold η ,

$$l \begin{matrix} H_1 \\ > \\ < \\ H_0 \end{matrix} \eta, \tag{1.6}$$

where

$$l = \frac{1}{N_0} \int_{T_1}^{T_f} z^*(t) h(t, \tau) z(\tau) dt d\tau, \tag{1.7}$$

and $h(t, \tau)$ satisfies the integral equation

$$N_0 h(t, \tau) + \int_{T_1}^{T_f} h(t, u) R(u, \tau) du = R(t, \tau), \quad T_1 \leq t, \tau \leq T_f. \tag{1.8}$$

The solution to (1.8) can also be written in terms of eigenfunctions and eigenvalues of the complex correlation function $R(t, \tau)$ [6],

$$h(t, \tau) = \sum_{i=1}^{\infty} \frac{\lambda_i}{\lambda_i + N_0/2} \phi_i(t) \phi_i(\tau), \quad T_1 \leq t, \tau \leq T_f. \tag{1.9}$$

One possible realization to generate l is the filter-correlator receiver see [5]. Even though the procedure is well defined, the actual implementation is difficult, mainly because of the dependence of $h(t, \tau)$ on λ_i and $\phi_i(t)$; $i=1, 2, \dots$. There are several categories of processes for which one can obtain a reasonably nearly

optimum solution [5, Ch. 4]. One of these categories is the low-energy-coherence (LEC) processes, in such cases the energy is distributed over large number of coordinates and for which all of the eigenvalues are small compared to the white noise one-sided power spectral density. It may appear that the LEC condition implies poor performance and is therefore uninteresting. This is not true because the receiver output is obtained by combining a large number of components, and the LEC condition can be met even though the ratio E_r/N_0 is larger than unity, where E_r is the average received energy over the entire observation interval. Under LEC conditions Eq. (7) reduces to

$$l = \frac{1}{N_0^2} \int_{T_1}^{T_2} \int z^*(t) R(t, \tau) z(\tau) dt d\tau. \quad (1.10)$$

The final question of interest is the performance of the optimum receiver under LEC condition. Van Tress [5, p. 136] showed that the probabilities of false alarm and detection are

$$P_F \approx Q\left(\frac{d}{2} + \frac{\eta}{d}\right) \quad (1.11)$$

$$P_D \approx Q\left(\frac{\eta}{d} - \frac{d}{2}\right), \quad (1.12)$$

respectively, where

$$d^2 \Delta \frac{\left[E\left\{ l \mid H_1 \right\} - E\left\{ l \mid H_0 \right\} \right]^2}{\text{Var}\left\{ l \mid H_0 \right\}}, \quad (1.13)$$

and

$$Q(x) = \int_x^\infty \frac{1}{2\pi} \exp\left[-\frac{x^2}{2}\right] dx. \quad (1.14)$$

Eqs. (1.11) and (1.12) are highly dependent on the scattering function of the channel through their dependence on d , therefore; in order to implement the optimum receiver and to evaluate its performance one needs to know the scattering function, $\sigma(r,f)$. Lack of complete knowledge of $\sigma(r,f)$ will result in degradation of the receiver's performance.

1.4 PERFORMANCE OF A MISMATCHED RECEIVER

Under mismatch conditions, the square root of the numerator in Eq. (1.13) can be written in terms of the true channel scattering function, $\sigma_t(r,f)$, and the assumed known to the receiver scattering function, $\sigma_a(r,f)$, as follows

$$E\left\{I | H_1\right\} E\left\{I | H_0\right\} = \frac{1}{N_o^2} \int \int \int \sigma_t(r,f) \sigma_a(r',f') | \chi(r-r', f-f') |^2 dr df dr' df' \quad (1.15)$$

and the denominator is

$$\text{Var}\left\{I | H_0\right\} = \frac{1}{N_o^2} \int \int \int \sigma_a(r,f) \sigma_a(r',f') | \chi(r-r', f-f') |^2 dr df dr' df', \quad (1.16)$$

where $\chi(\tau,\nu)$ is the ambiguity function of the complex envelope of the transmitted signal, namely

$$\chi(\tau,\nu) = \int_{-\infty}^{\infty} u(t) u^*(t-\tau) \exp [j2\pi\nu t] dt. \quad (1.17)$$

Case 1: Scattering Functions With Unknown Parameters

The first case under consideration is the case when there is a mismatch in the mean time delay and the mean Doppler shift by say x_1 and x_2 respectively. Then the true and the assumed scattering functions can be related as follows

$$\sigma_a(r,f) = \sigma_t(r-x_1, f-x_2) \quad (1.18)$$

Substituting in Eqs. (1.15) and (1.16), we obtain

$$d^2 = \frac{1}{N_o^2} \frac{\left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_a(r, f) \sigma_a(r', f') | \chi(r-r' + x_1, f-f' + x_2) |^2 dr df dr' df' \right]^2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_a(r, f) \sigma_a(r', f') | \chi(r-r', f-f') |^2 dr df dr' df'} \quad (1.19)$$

In order to proceed further with the calculations, one needs to specify the shape of the true scattering function and the envelope of the transmitted waveform. If the true scattering function of the channel is assumed to have the following shape

$$\sigma_t(r, f) = \frac{E_r}{2\pi LB} \exp \left[-r^2/2L^2 - f^2/2B^2 \right], \quad (1.20)$$

where E_r is the total received energy when the complex envelope of the transmitted waveform has unit energy. And the envelope of the transmitted signal is

$$u(t) = \begin{cases} 1/\sqrt{T} & 0 \leq t \leq T \\ 0 & \text{elsewhere.} \end{cases} \quad (1.21)$$

Then the true autocorrelation function is

$$R(t, \tau) = \begin{cases} E_r/T \exp \left[-2(\pi B(\tau-t))^2 \right] \left\{ Q(\max(\tau-T)/L) - Q(\min(t, \tau)/L) \right\} & |\tau-t| \leq T \\ 0 & \text{elsewhere.} \end{cases} \quad (1.22)$$

Equation (1.19) can be evaluated numerically. Then the probabilities of detection, P_D , and false alarm, P_F , can be determined by substituting in Equations (1.11) and (1.12). Figures 1.2 and 1.3 demonstrate the change in the probability of detection, P_D , versus SNR for probability of false alarm, P_F , of 10^{-4} and different values of mean time delay, x_1 , and the mean frequency shift, x_2 . It is clear that the receiver is more sensitive to misalignment in frequency shifts than misalignments in time delays. Also, the amount of degradation in performance for

under-spread channels ($s=0.1$) for $x_2 > B$ and $x_1 > 5L$ is severe and the system designer should provide the adequate circuitry to insure synchronization.

Case 2: Completely Unknown Scattering Functions

If the shape of the true scattering function is not known, the receiver has, either to estimate it or use an approximation for $\sigma_t(r, f)$. In both cases the true and the assumed scattering functions may differ in shape and the resulting measure of performance, d^2 , is

$$d^2 = \frac{1}{N_o^2} \frac{\left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_t(r, f) \sigma_a(r', f') | \chi(r-r', f-f') |^2 dr df dr' df' \right]^2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_a(r, f) \sigma_a(r', f') | \chi(r-r', f-f') |^2 dr df dr' df'}. \quad (1.23)$$

For the present discussion, $\sigma_a(r, f)$ will be taken as

$$\sigma_a(r, f) = \begin{cases} E_r/LB & |f| \leq B/2, 0 \leq r \leq L \\ 0 & \text{elsewhere,} \end{cases} \quad (1.24)$$

while the true scattering function, $\sigma_t(r, f)$ will be represented by Eq. (1.20).

Figure 1.4 illustrates the effect of mismatch in the scattering functions for this specific case.

For over-spread channels, ($s > 10.0$), equation (1.22) is very closely approximated by a knife-edge (ridge) function along the $t-\tau$ axis. In this case we expect that the system become more sensitive for any time delays, frequency shifts, or mismatch in the shape of the scattering functions.

It is noticed also that higher values of P_F will not change the previous results significantly.

1.5 CONCLUSIONS

In order to implement the optimum receiver over fading dispersive channels in practice, one needs the filter impulse response $h(t, \tau)$. This is obtained by solving the integral equation (8) which depends on the scattering function $\sigma(r, f)$ through the autocorrelation function $R(t, \tau)$. Both measurement of the $\sigma(r, f)$ and the solution of the integral equation present barriers to the actual receiver implementation.

This triggered the need to study the sensitivity of a mismatched receiver. We began by studying the effects of mismatch in the location of the scattering function. It has been demonstrated that for a Gaussian shaped scattering functions and under-spread channels, the amount of degradation is severe if $x_1 > 5L$ and $x_2 > B$. We next studied the degradation due to mismatch in the shape of the scattering functions. A degradation of about 1 dB is encountered if a rectangular scattering function instead of a Gaussian shaped scattering function is used at the receiver, provided that the true scattering function was Gaussian.

It worthwhile mentioning that if we change the role of σ_t and σ_s the amount of the degradation will not be the same. As a matter of fact, it can be shown that if $\sigma_s(r, f)$ is Gaussian but $\sigma_t(r, f)$ is rectangular, then the degradation in performance is less severe.

For over-spread channels, the autocorrelation functions, $R(t, \tau)$ tend to concentrate around the $t = \tau$ axis and any misalignment in location or mismatch in shape will have a greater degradation effects on the performance of the receiver.

The study emphasizes the fact the synchronization over fading dispersive channels is important and in some cases the degradation due to missynchronization may be more severe than mismatch in the shape of the scattering function, this mismatch may be a result of error in measuring of the scattering function.

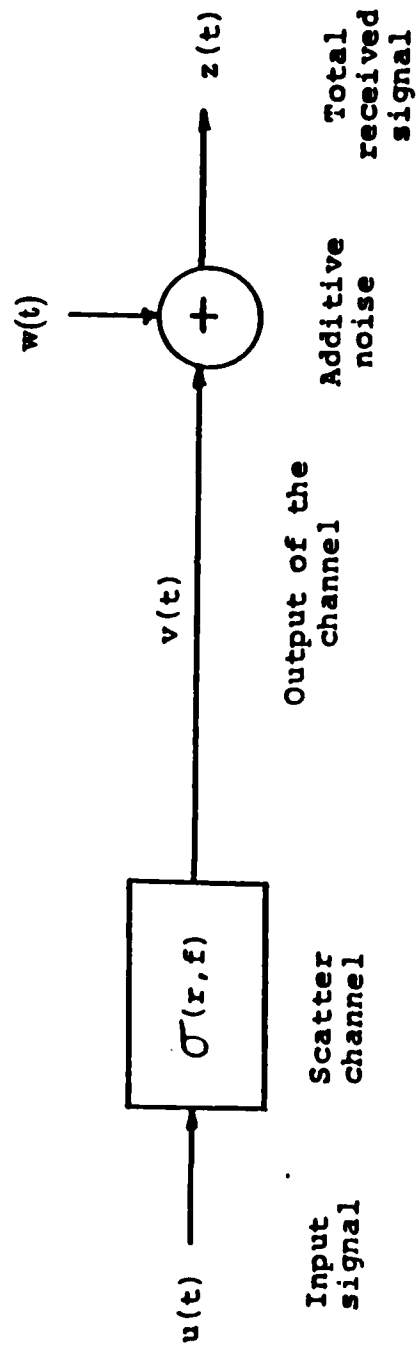


Figure 1.1 The Channel Model

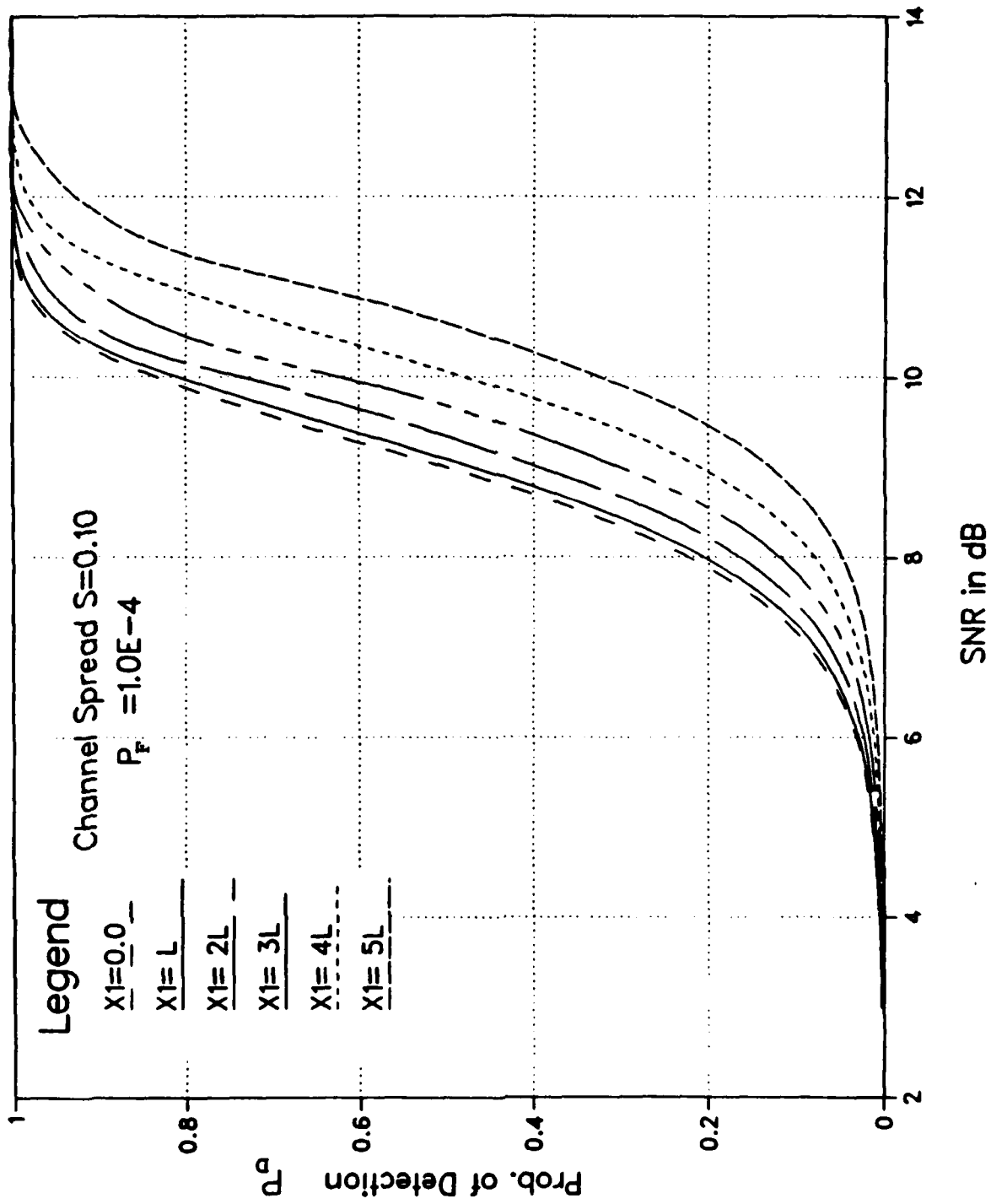


Figure 1.2 Mismatch in the Location of the Scattering Function

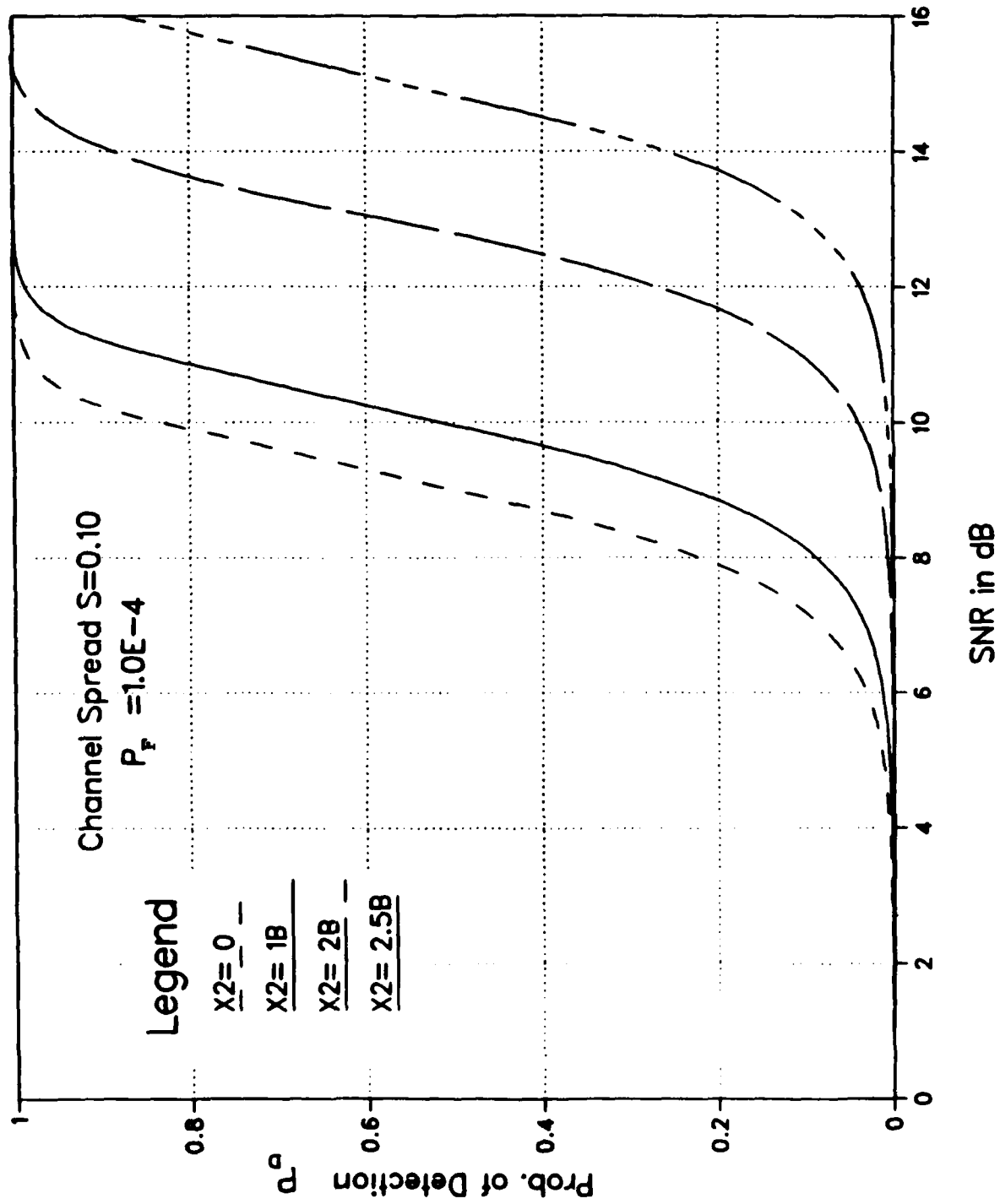


Figure 1.3 Mismatch in the Location of the Scattering Function

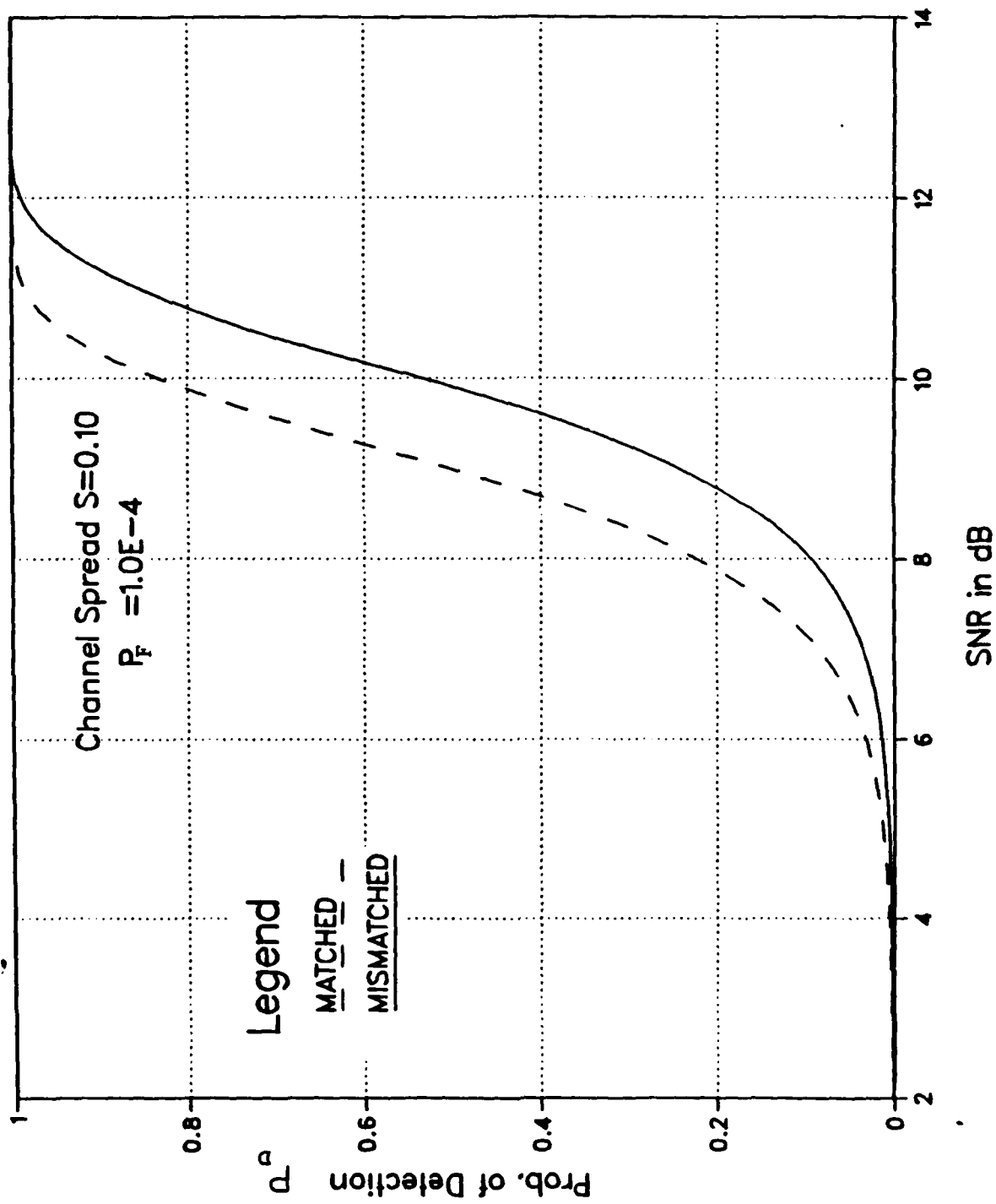


Figure 1.4 Effect of Mismatch in Scattering Functions

CHAPTER II

SERIAL SEARCH SYNCHRONIZATION OVER FADING DISPERSIVE CHANNELS

Although; the engineering importance of fading dispersive channels has increased markedly in the recent years. The problem of synchronization over such channels has not been given the attention it deserves. The following analysis derives and evaluates upper and lower bounds on the mean and variance of synchronization time for a very general serial search system. The objective is to find a simple result that the system designer will be able to use to make design trade-off studies to minimize mean synchronization time.

It has been found that the results are highly dependent on the scattering function of the channel, the degree of the channel spread, number of the cells used in the search procedure, and the the time required to reject an incorrect cell when a false alarm occurs.

2.1 INTRODUCTION

Synchronization is a fundamental problem in digital communication, radar, sonar, and navigation systems. Power-efficient receivers generally require the existence of a clock that is accurately time-aligned with the received pulses, and a local carrier reference, that agrees closely in frequency and phase with the received carrier. In general terms, two sequences of events are said to be synchronous if the corresponding events in the two sequences occur simultaneously, with one of the two sequences of events takes place at the transmitter, and the other takes place at the receiver. Due to transmitter oscillator instability and propagation effects, the two sequences of events may be misaligned.

Losing synchronization reduces the efficiency in the data detection process; because the inaccurate symbol sync directly reduces the probability of making correct decisions. Moreover, when a loss of synchronization occurs, it may sometimes lead to successive errors before it is regained; these successive errors affect the overall performance of the system.

Synchronization is defined simply as the process of bringing about, and retaining, a synchronous situation. It is generally convenient to separate the synchronization process into two distinct modes. In the first mode, the clock synchronization mode, the clocks which regulate the two sequences being synchronized (i.e., the transmitter and receiver clocks) are forced to run at the same rate. In the second mode, the higher order synchronization mode, a corresponding pair of events in the two sequences are identified and made to occur simultaneously, and if the sequences are progressing at the same rate, the sequences are, and will remain, synchronized.

If the transmitter and the receiver clocks are both sufficiently stable relative to the required synchronization accuracy, the clock synchronization mode may be by-passed. Most generally, however, this will not be the case and some technique must be devised to provide the needed clock synchronization.

Once the transmitter and receiver clocks have been synchronized the second mode of the synchronization process begins, this includes symbol, code word, data word, and frame synchronization. The second mode can be further subdivided into two components, the first component is the determination of the initial parameters from whatever a priori information available. This component is known as the acquisition phase. The second component is to maintain synchronization after initial acquisition. This problem is known as the tracking phase.

Although; the problem of synchronization over additive white Gaussian noise (AWGN) channels has been investigated extensively [7-10], synchronization over fading dispersive channels has not been given the attention it deserves, in spite of the fact that the the engineering importance of such channels has increased markedly in the recent years. Fading dispersive channels are usually best described as random linear time-varying filters . In many applications it is reasonable to suppose that the impulse response of the filter is a sample function of a Gaussian random process.

In most of the work done concerning detection over fading dispersive channels, the assumption that synchronization is available is either understood implicitly or declared explicitly.

2.2 SYNCHRONIZER STRUCTURE

In communicating over fading dispersive channels, the received energy is peaked at some point P (see Figure. 2.1), which is shifted from the transmitter clock pulse and oscillator frequency, point T, by \bar{r} sec. in time and \bar{f} Hz in frequency, where \bar{r} and \bar{f} are the mean time delay and mean frequency shift of the channel respectively. All the points are referred to the receiver clock pulse and oscillator frequency which are located at the origin of the figure. It is clear that no matter what offset exists between the receiver R and the transmitter T, the important information to the receiver is the vector \underline{x} , where $\underline{x} = (x_1, x_2)$. Therefore the main objective of the receiver is to come up with a good estimate of the vector \underline{x} .

The vector \underline{x} is the misalignment in time and frequency between the receiver and transmitter is the result of two effects, the first is the contribution of the channel, and the second is due to the lack of synchronization between the

receiver's and transmitter's clock pulses and oscillator frequencies. This lack of synchronization always exists initially even if the channel introduces no delay in time or shift in frequency. Since these two effects are of the same nature they can be lumped together into the mean values of the scattering function. therefore from now and on we will assume that the scattering function is of the form

$$\sigma(r, f; \underline{x}) = \sigma(r - x_1, f - x_2). \quad (2.1)$$

For such scattering function the correlation function of the complex envelope of the output of the channel for single pulse is

$$R(t, \tau; \underline{x}) = R_o(t - x_1, \tau - x_1) \exp \left[j 2 \pi x_2 (\tau - t) \right] \quad (2.2)$$

where $R_o(t, \tau)$ is the complex correlation function of the complex envelope of the output of the channel in the case of perfect synchronization.

The theory of maximum likelihood (ML) estimation provides us with a technique to estimate the channel parameters needed for the sync procedure. In [4] we derived the likelihood function. The rule is \underline{x} is more likely than \underline{x}' iff

$$W(\underline{x}) > W(\underline{x}'),$$

where

$$W(\underline{x}) = \int \int z^*(t) h(t, \tau; \underline{x}) z(\tau) dt d\tau. \quad (2.3)$$

$h(t, \tau; \underline{x})$ is the solution of the following integral equation

$$\int R(t, u; \underline{x}) h(u, \tau; \underline{x}) du + N_o h(t, \tau; \underline{x}) = R(t, \tau; \underline{x}). \quad (2.4)$$

$N_o/2$ is the two sided power spectral density of the AWGN, $z(t)$ is the complex envelope of the received waveform (see Figure 1.1).

Under low-energy-coherence condition [5] equation (2.3) is reduced to

$$W(\underline{x}) = \int \int z^*(t) R(t, \tau; \underline{x}) z(\tau) dt d\tau. \quad (2.5)$$

The precise implementation of the maximum likelihood estimator would involve a continuum of detectors (or calculations) for each value of \underline{x} in the region of interest. Certainly, in general such estimator cannot be built. One realizable approximation to the maximum likelihood estimator results from specifying a finite set of points, say $\{ \underline{x}_i, i=1, \dots, N \}$, and record $z(t)$ then perform the parallel processing operation shown in Figure 2.2 in which $h(t, \tau; \underline{x})$ is viewed as a linear time-varying filter with τ as the input time variable and t as the output time variable. The output of this processor is N numbers, each is an output of a detector matched to a particular \underline{x} . The ML estimator finds the largest detector output and assumes that the correct value of \underline{x} is the one corresponding to that detector. The result is a parallel processor, in each parallel branch the likelihood function is evaluated and the maximum is chosen to represent the maximum likelihood estimate of the vector of parameters under consideration. This synchronizer which is optimum in the sense that it achieves synchronization with a given probability in the minimum possible time requires a detector for every cell and thus is not optimum in a minimum hardware sense. Thus minimum acquisition time systems is never implemented because of excessive hardware complexity. A synchronization system that evaluates the cells serially until the correct cell is found is said to use serial search. Such systems are designed to achieve a compromise between acquisition time and reasonable complexity without compromising any other important system characteristic. The performance of such serial search synchronizers are analyzed in the next section.

2.3 PERFORMANCE ANALYSIS

If the uncertainty region is divided into N cells as shown in Figure 2.3, where the size of each cell is proportional to the width of the autocorrelation ambiguity function. This function is the expected value of the random variable $W(\mathbf{x})$. A complete discussion of the properties of this autocorrelation ambiguity function is given in [11].

Because it is equally likely that the correct vector is in any cell, the search can begin at any corner of the uncertainty region. The search will advance through one cell at a time until N cells have been evaluated. If synchronization has not been achieved at that time, a retrace will start the search over again at the starting position.

The mean synchronization time is calculated by considering all possible sequences of events leading to a correct synchronization. An event in the probability space being considered is defined by a particular location, n , for the correct cell, a particular number of missed detection, j , of the correct cell, and a particular number of false alarms, k , in all incorrect cells evaluated. The total synchronization time for a particular event defined by (n,j,k) is

$$T(n,j,k) = nT_e + jNT_e + kT_{fa}, \quad (2.6)$$

where T_e is the evaluation time for each cell, and T_{fa} is the time required to reject an incorrect cell when a false alarm occurs. It can be shown that the probability of the event (n,j,k) is

$$\text{pr}(n,j,k) = \frac{1}{N} p_d (1-p_d)^j \binom{K}{k} p_{fa}^k (1-p_{fa})^{K-k}. \quad (2.7)$$

The mean synchronization time is

$$\bar{T}_s = \sum_{n,j,k} T(n,j,k) \text{pr}(n,j,k). \quad (2.8)$$

After some straightforward algebraic manipulation [30-Ch. 10], equation (12) is reduced to

$$\bar{T}_s = (N-1)T_{ad}\left(\frac{2-p_d}{2p_d}\right) + \frac{T_e}{p_d}, \quad (2.9)$$

where

$$T_{ad} = T_e + T_{fa}p_{fa}, \quad (2.10)$$

is the average dwell time on each cell. Equation (2.9) also, has been derived in [12] using signal flow graph techniques. It can be shown also, that the variance of the synchronization time is approximated by

$$\sigma_{T_s}^2 \approx T_{ad}^2 N^2 \left(\frac{1}{12} - \frac{1}{p_d} + \frac{1}{p_d^2} \right) \quad (2.11)$$

The approximation is valid for $N \gg 1$, $p_{fa} \ll 1$, and $1-p_d \ll 1$.

Calculations of p_d and p_{fa} :

The serial search synchronizer under consideration, evaluates each cell by estimating whether or not signal energy is present at the output of each processor in Figure 2.2 or simply

$$\begin{matrix} H_1 \\ W(\underline{x}_i) > \gamma \\ H_0 \end{matrix} \quad (2.12)$$

where, H_1 is the hypothesis that the \underline{x}_i is the correct \underline{x} , while H_0 is the hypothesis that \underline{x}_i is not the correct cell. This problem is identical to the problem of detection over fading dispersive channels which is well treated in [3] and [5]. Van Trees [5, Ch. 4] showed that

$$p_d \approx Q\left(\frac{\gamma}{d} - \frac{d}{2}\right) \quad (2.13)$$

$$p_{fa} \approx Q\left(\frac{\gamma}{d} + \frac{d}{2}\right), \quad (2.14)$$

where

$$d^2 \Delta \frac{(\mathbb{E}\{W | H_1\} - \mathbb{E}\{W | H_0\})^2}{\text{Var}\{W | H_0\}}, \quad (2.15)$$

and

$$Q(x) = \int_x^\infty \frac{1}{2\pi} \exp\left[-\frac{x^2}{2}\right] dx. \quad (2.16)$$

In order to proceed further with the calculations, one needs to know the shape of both the channel scattering function and the complex envelope of the transmitted wave form. Assuming Gaussian-shaped functions

$$\sigma(r, f; \underline{x}) = \frac{E_r}{2\pi LB} \exp\left[-(r-x_1)^2/2L^2 - (f-x_2)^2/2B^2\right], \quad (2.17)$$

where E_r is the total received energy when the complex envelope of the transmitted waveform has a unit energy. The complex envelope of the transmitted waveform is

$$u(t) = (2\frac{\alpha}{\pi})^{1/4} \exp\left[-\alpha t^2\right]. \quad (2.18)$$

It has been shown that [4] the optimum value of α is

$$\alpha = \pi \frac{B}{L} \quad (2.19)$$

Figures 2.4 and 2.5 demonstrate how p_d behave as a function of the signal-to-noise ratio (SNR) and p_{fa} for both under-spread, and over-spread channels respectively. These results are valid only under LEC condition, and if the width of each cell is greater than or equal to the width of the autocorrelation ambiguity function in both time and frequency. This put a restriction on the

accuracy of the estimate, and the number of cells N

$$N \leq \frac{T\Omega}{\Delta\Gamma} \quad (24)$$

where Δ and Γ are the width of the autocorrelation in time and frequency respectively.

2.4 CONCLUSIONS

A method to estimate the mean synchronization time for synchronization over fading dispersive channels has been shown. Equation (13) demonstrates the dependence of the mean synchronization time, \bar{T}_s , on p_d , p_{fa} , T_e , T_{fa} , and N . To minimize \bar{T}_s , one has to select the optimum set of parameters. Some of these parameters, as N , can be controlled by the designer, the remaining, p_d , p_{fa} , and T_e are dependent on the shape of the scattering function, the amount of spread of the channel, and SNR.

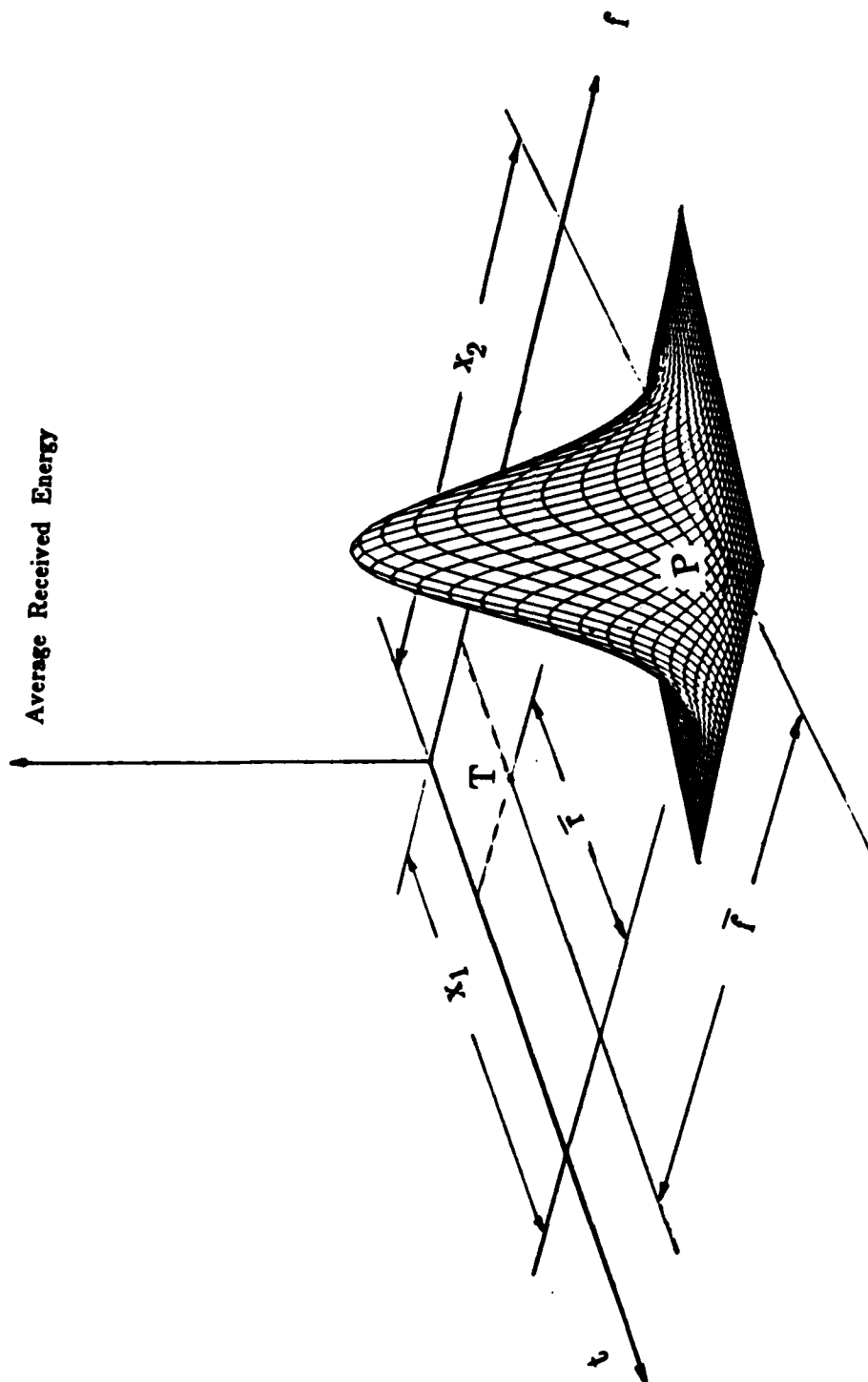


Figure 2.1 Combined Effect of the Channel Propagation
and the Transmitter-Receiver Offset

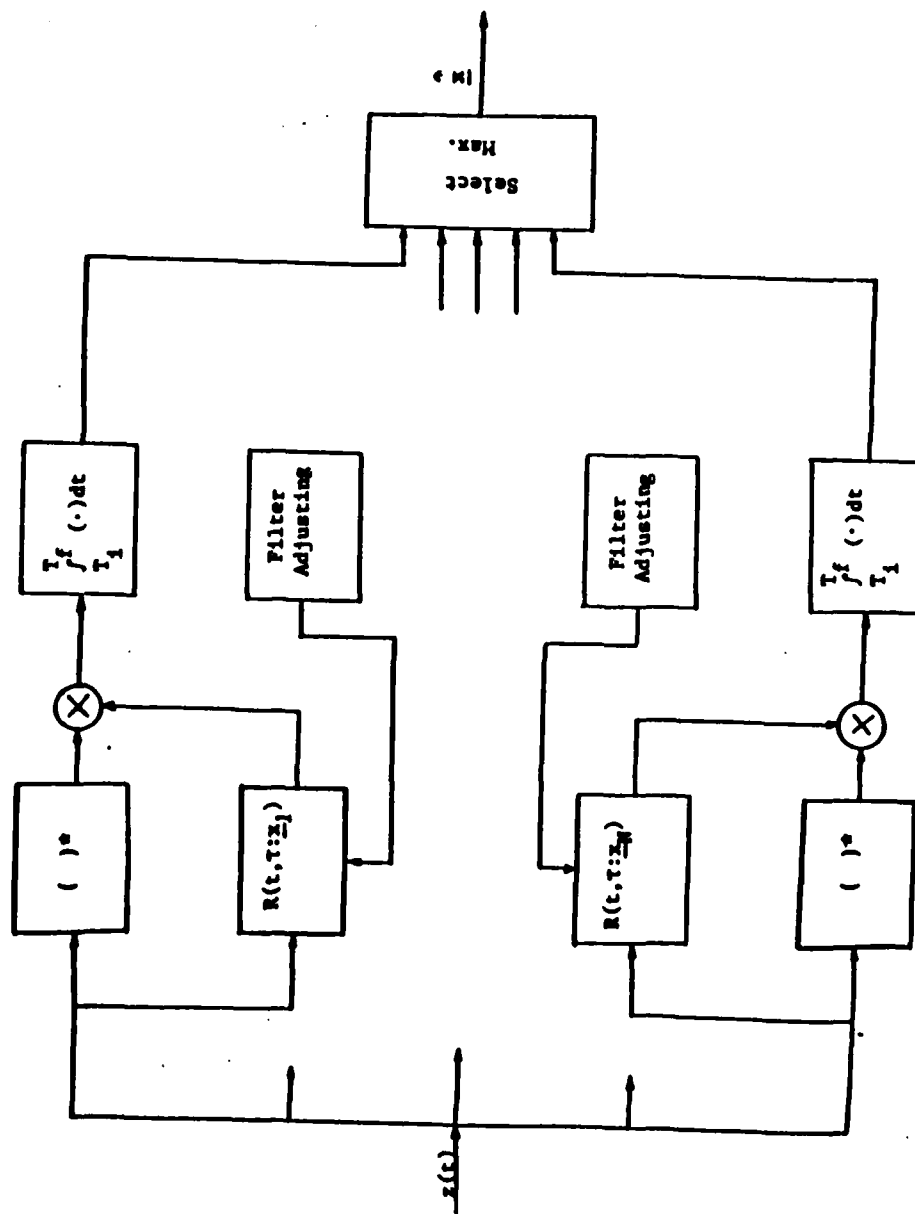


Figure 2.2 Synchronizer Structure (Parallel Processor)

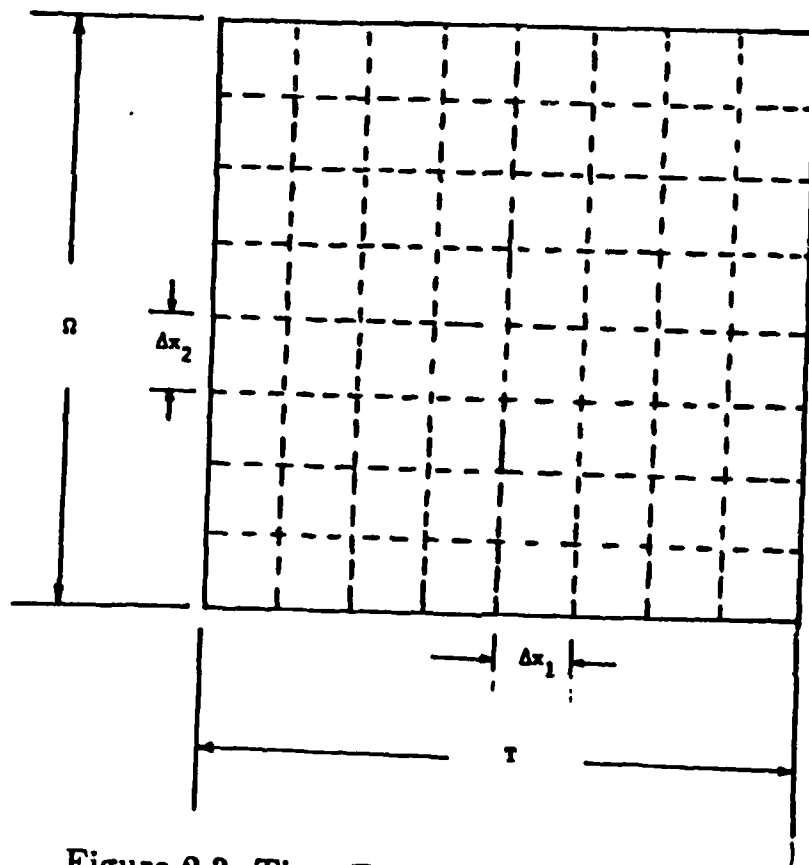
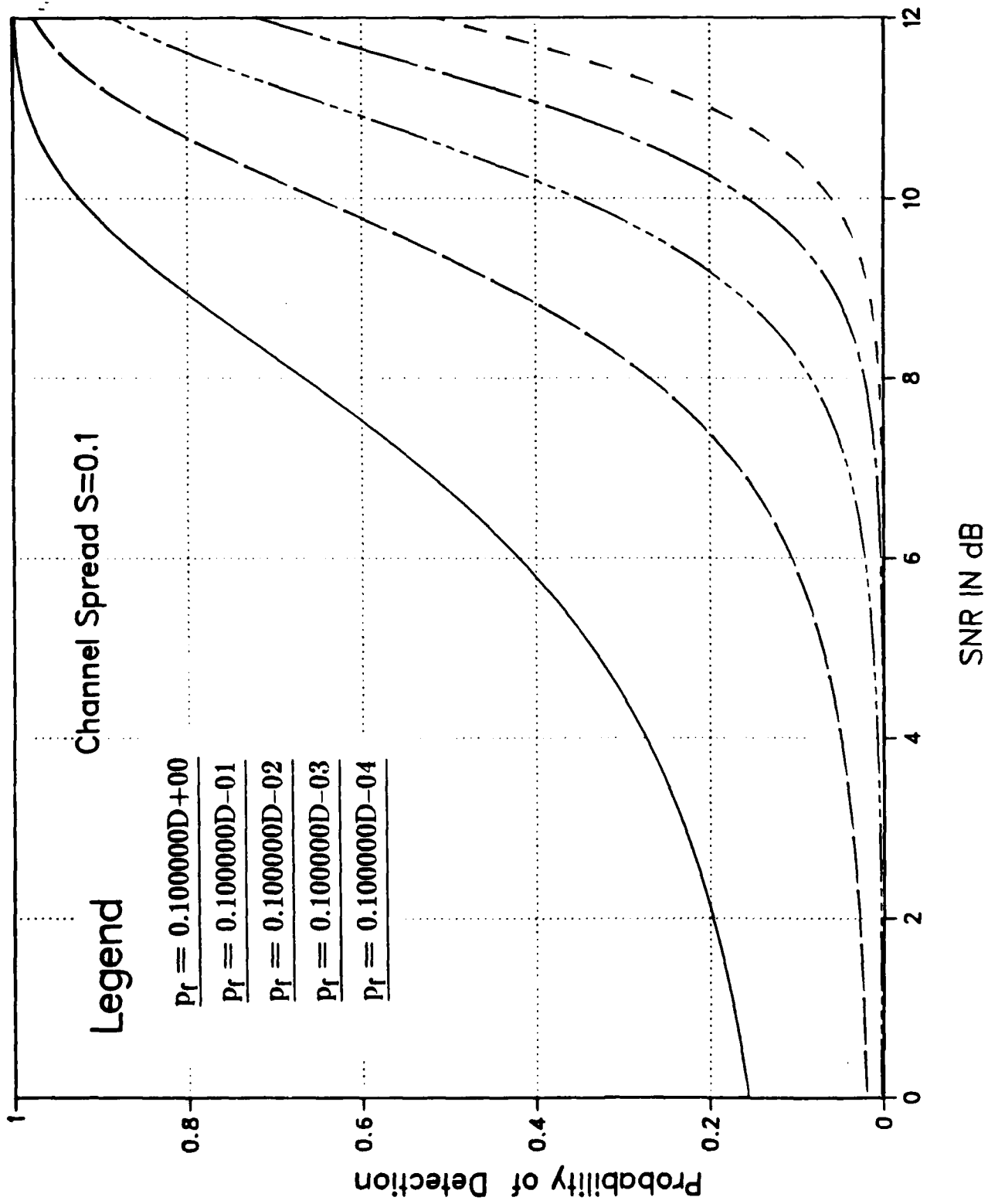
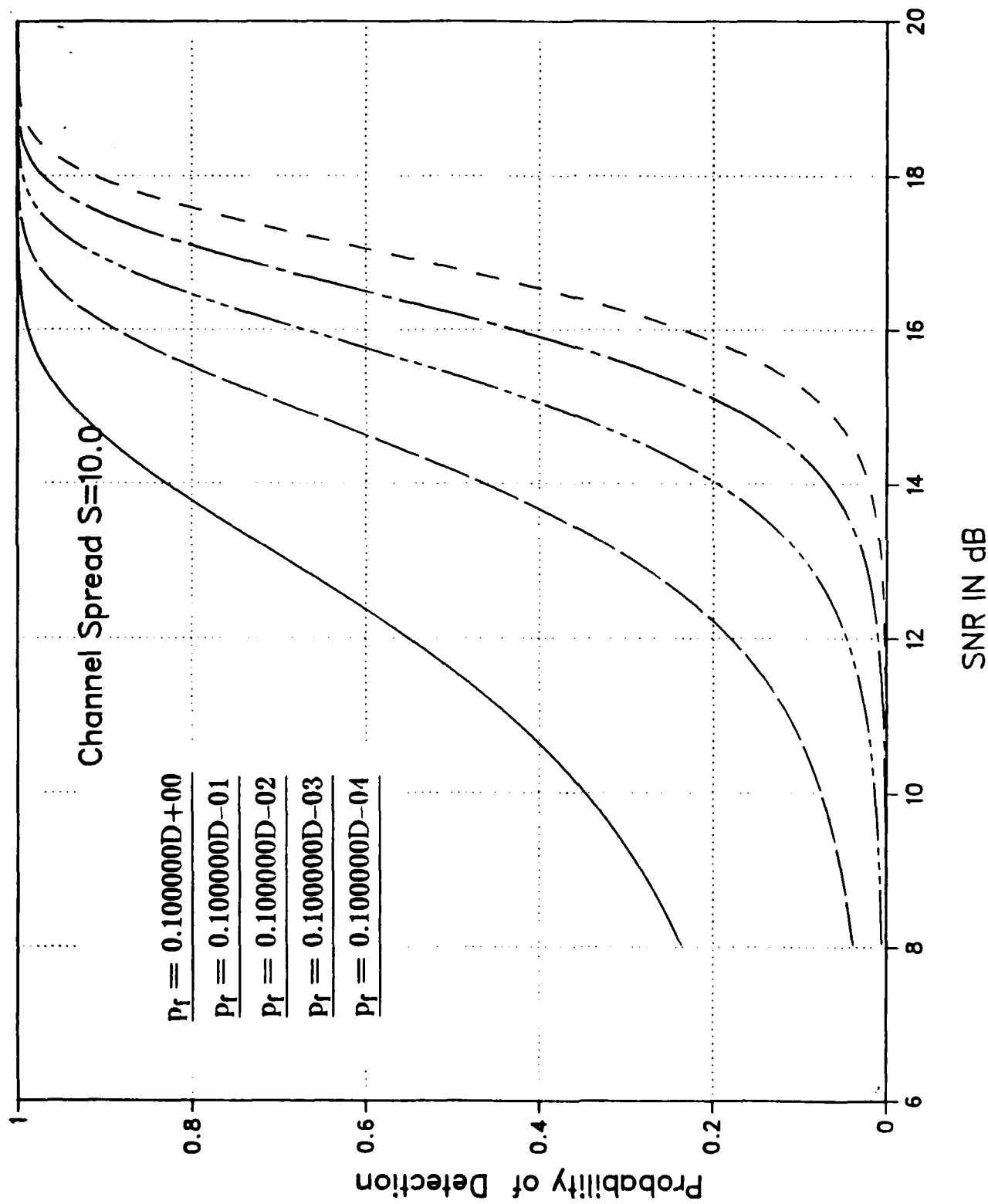


Figure 2.3 Time Frequency Uncertainty Region

Figure 2.4 p_d as a Function of SNR and p_f

Figure 2.5 p_d as a Function of SNR and p_f

CHAPTER III

CLOSED LOOP SYNCHRONIZER OVER FADING DISPERSIVE CHANNELS

In this chapter an optimum closed loop structure for symbol synchronizer over fading dispersive channels is developed. The optimum synchronizer is similar to those already known symbol synchronizers that are being used over additive white Gaussian noise (AWGN) channels, except that the control signal is completely random; even in the absence of the AWGN. The reason is clear, and simply because of the structure of the received signal over fading dispersive channels. It is shown that the synchronizer structure is highly dependent on the scattering function of the channel. For a Gaussian shaped scattering function, a simple closed loop structure is obtained and expressions for tracking error statistics are derived. The dependency of such statistics on the signal-to-noise ratio and on the spread of the channel is investigated.

3.1 INTRODUCTION

It is generally convenient to separate the synchronization process into two distinct modes. In the first mode, the clock synchronization mode, the clocks which regulate the two sequences being synchronized (i.e., the transmitter and receiver clocks) are forced to run at the same rate. In the second mode, the higher order synchronization mode, a corresponding pair of events in the two sequences are identified and made to occur simultaneously, and if the sequences are progressing at the same rate, the sequences are, and will remain, synchronized.

If the transmitter and the receiver clocks are both sufficiently stable

relative to the required synchronization accuracy, the clock synchronization mode may be by-passed. Most generally, however, this will not be the case and some technique must be devised to provide the needed clock to maintain synchronization after initial acquisition. This problem is known as the tracking phase.

The advent of highly compact, inexpensive digital computers has now made it possible to exploit well-known results from statistical estimation theory and control theory to develop systems that automatically respond to changing signal environment. This self adjusting or adaptive capability renders the operation of such systems more flexible, reliable, and more importantly offers improved reception performance that would be difficult to achieve in any other way. These reasons motivate the idea of developing a synchronizer which updates itself (closed loop structure).

In the following sections, a closed loop structure synchronizer is derived and its performance is investigated.

As in chapter II, the vector \underline{x}_t is used to indicate the misalignment in time and frequency between the receiver and transmitter is the result of two effects, the first is the contribution of the channel, and the second is due to the lack of synchronization between the receiver's and transmitter's clock pulses and oscillator frequencies. This lack of synchronization always exists initially even if the channel introduces no delay in time or shift in frequency. Since these two effects are of the same nature they can be lumped together into the mean values of the scattering function. This will result in scattering functions as described by equation (2.1). The optimum estimate of \underline{x} is such that

$$W(\underline{x}) > W(\underline{x}') \quad (3.1)$$

where

$$W(\underline{x}) = \frac{1}{N_0^2} \iint z^*(t) h(t, \tau; \underline{x}) z(\tau) dt d\tau, \quad (3.2)$$

$h(t, \tau; \underline{x})$ is the solution of the following integral equation given by equation (2.4), and $N_o/2$ is the two sided power spectral density of the AWGN, and $z(t)$ is the complex envelope of the received waveform (see Figure 1.1). Under low-energy-coherence condition [5] equation (3.2) is reduced to

$$W(\underline{x}) = \frac{1}{N_o^2} \iint z^*(t) R(t, \tau; \underline{x}) z(\tau) dt d\tau. \quad (3.3)$$

The low-energy-coherence condition is more realistic for over-spread channels.

In the case of repeated transmission the transmitted waveform can be written as

$$s(t) = \sum_{i=1}^K s_i(t - (i-1)/\rho) \quad (3.4)$$

where ρ is the transmission rate. It can be shown that [4] the likelihood function for estimating \underline{x}_t in the case of repeated transmission is

$$W(\underline{x}) = \frac{1}{N_o^2} \sum_{i=1}^K \iint z_i^*(t) R_i(t, \tau; \underline{x}) z_i(\tau) dt d\tau, \quad (3.5)$$

where $R_i(t, \tau; \underline{x})$ is the complex correlation function associated with the i -th transmitted waveform.

As mentioned before, the precise implementation of the maximum likelihood estimator would involve a continuum of detectors (or calculations) for each value of \underline{x} in the region of interest. Certainly, in general such estimator cannot be built. One realizable approximation to the maximum likelihood estimator results from specifying a finite set of points, say $\{ \underline{x}_i, i=1, \dots, N \}$, and record $z(t)$ then perform the parallel processing operation in which $h(t, \tau; \underline{x})$ is viewed as a linear time-varying filter with τ as the input time variable and t as the output time variable. The output of this processor is N numbers, each is an output of a detector matched to a particular \underline{x} . The ML estimator finds the largest detector output

and assumes that the maximum likelihood estimate of \underline{x}_t is the one corresponding to that detector. The result is a parallel processor, in each parallel branch the likelihood function is evaluated and the maximum is chosen to represent the maximum likelihood estimate of the vector of parameters under consideration. This synchronizer which is optimum in the sense that it achieves synchronization with a given probability in the minimum possible time requires a detector for every cell and thus is not optimum in a minimum hardware sense. Thus minimum acquisition time systems is never implemented because of excessive hardware complexity. A synchronization system that evaluates the cells serially until the correct cell is found is said to use serial search. Such systems are designed to achieve a compromise between acquisition time and reasonable complexity. The performance of a serial synchronizer is discussed in [13].

In the following sections a closed loop synchronizer is derived and its performance is analyzed.

3.2 CLOSED LOOP STRUCTURE

Our objective is a closed loop structure that can be derived from the ML equations. The reluctance to develop a closed loop structure for the synchronizer is due principally to the continuously changing nature of the characteristics of the channel. The technique we will use to transform the open loop structure (parallel processor) derived from the maximum likelihood theory to the desired closed loop structure is the gradient approach. The gradient approach is very popular since it is a relatively simple and generally well understood method that permits the solution of a large class of problems. When the likelihood function is near quadratic, then the performance measure can be visualized as a bowl-shaped surface, so the synchronizer has the task of continually seeking the "top

of the bowl". In this case, seeking the maximum of the likelihood function can be accomplished by "hill climbing" method of which the various gradient methods are representative. For the ML function (3.5), the gradient is obtained by differentiating with respect to the vector \underline{x} to yield

$$\frac{\partial W(\underline{x})}{\partial x_j} = \sum_{i=1}^K \int \int z_i^*(t) \frac{\partial R_i(t, \tau; \underline{x})}{\partial x_j} z_i(\tau) dt d\tau, \quad j=1,2 \quad (3.6)$$

In order to proceed further with the analysis, one needs to know the shape of the channel scattering function. By assuming a Gaussian-shaped scattering function of the form

$$\sigma(r, f; \underline{x}) = \frac{E_r}{2\pi LB} \exp \left[-(r-x_1)^2/2L^2 - (f-x_2)^2/2B^2 \right], \quad (3.7)$$

where E_r is the total received energy when the complex envelope of the transmitted waveform has unit energy. Using such scattering function, equations (12) can be reduced to

$$\frac{\partial W(\underline{x})}{\partial x_j} = x_j W(\underline{x}) - \sum_{i=1}^K \int \int z_i^*(t) R_{ij}(t, \tau; \underline{x}) z_i(\tau) dt d\tau, \quad j=1,2 \quad (3.8)$$

where

$$R_{i1}(t, \tau; \underline{x}) = \int \int r \sigma(r, f; \underline{x}) u_i(t-r) u_i^*(\tau-r) \exp \left[j2\pi f(\tau-t) \right] dr df, \quad (3.9-a)$$

and

$$R_{i2}(t, \tau; \underline{x}) = \int \int f \sigma(r, f; \underline{x}) u_i(t-r) u_i^*(\tau-r) \exp \left[j2\pi f(\tau-t) \right] dr df. \quad (3.9-b)$$

Equating (3.9-a) and (3.9-b) to zero and solving the two transcendental equations simultaneously will yield the maximum likelihood estimate of \underline{x} .

An alternative to be considered in this paper is to use the partial derivatives

$$\frac{\partial w(\underline{x})}{\partial x_j}, \quad j=1,2 \quad (3.10)$$

to control the current estimate of \underline{x} . Since this expression will be zero when \underline{x} equals the true vector \underline{x}_t , and since it is monotonically increasing function of \underline{x} in the vicinity of $\underline{x}=\underline{x}_t$, a feedback device should be able to force \underline{x} to converge to the maximum likelihood estimate of \underline{x}_t . The resulting symbol-tracking device is shown in Figure 3-1. The output of the box labeled "accumulator" is

$$e_j = x_j W(\underline{x}) - W_j(\underline{x}), \quad j=1,2; \quad (3.11)$$

where

$$W_j(\underline{x}) = \sum_{i=1}^K \int \int z_i^*(t) R_{ij}(t, \tau; \underline{x}) z_i(\tau) dt d\tau, \quad j=1,2; \quad (3.12)$$

The error signals e_j , $j=1,2$ are random variables, their means and variances are given by

$$E\{e_j\} = x_j E\{W(\underline{x})\} - E\{W_j(\underline{x})\} \quad (3.13)$$

$$\text{Var}\{e_j\} = x_j^2 \text{var}\{W(\underline{x})\} + \text{var}\{W_j(\underline{x})\} - 2x_j \text{cov}\{W, W_j\} \quad (3.14)$$

3.3 NUMERICAL RESULTS

To evaluate the different variances one needs to specify the complex envelope of the transmitted signal. For Gaussian-shaped scattering function, it has been found that a Gaussian-shaped envelope in the form

$$u(t) = (2\alpha/\pi)^{1/4} \exp[-\alpha t^2], \quad (3.15)$$

is optimum from the detection point of view [4]. Other reasons for the choice of (3.7) and (3.15) for the forms of the scattering function and the complex envelope of the transmitted waveform are based on the fact that the Gaussian function

1. is a simple and smooth function,
2. has elegant properties when integrated against another Gaussian function on the infinite interval,
3. can be used to approximate many of the finite-duration pulse signals commonly used in communications, and
4. is easily generated [14].

It has been shown that the value of α in (3.15) which minimizes the area of the uncertainty ellipse [4] is

$$\alpha = \pi \frac{B}{L}. \quad (3.16)$$

The statistics of $W(\underline{x})$ and W_j are

$$E\{W\} = \sum_{i=1}^K \left[\text{SNR} + \frac{1}{N_0^2} \iint R_i^*(t, \tau; \underline{x}_t) R_i(t, \tau; \underline{x}) dt d\tau \right] \quad (3.17)$$

$$E\{W_j\} = \sum_{i=1}^K \left[x_j \text{SNR} + \frac{1}{N_0^2} \iint R_i^*(t, \tau; \underline{x}_t) R_{ij}(t, \tau; \underline{x}) dt d\tau \right], \quad (3.18)$$

$$\begin{aligned} \text{var}\{W\} = & \sum_{i=1}^K \left[\frac{1}{N_0^4} \iiint \iint R_i^*(t, \tau_1; \underline{x}_t) R_i(t, \tau; \underline{x}) R_i^*(t_1, \tau_1; \underline{x}_t) R_i(t_1, \tau; \underline{x}) dt d\tau dt_1 d\tau_1 \right. \\ & + \frac{1}{N_0^3} \iint \iint R_i^*(t, \tau_1; \underline{x}_t) R_i(t, \tau; \underline{x}) R_i(\tau, \tau_1; \underline{x}) dt d\tau d\tau_1 \\ & + \frac{1}{N_0^3} \iint \iint R_i^*(t_1, \tau_1; \underline{x}_t) R_i(t_1, t; \underline{x}) R_i(t, \tau; \underline{x}) dt d\tau dt_1 \\ & \left. + \frac{1}{N_0^2} \iint R_i^*(t, \tau; \underline{x}) R_i(t, \tau; \underline{x}) dt d\tau \right] \quad (3.19) \end{aligned}$$

the quantity $\text{var}\{W_j\}$ has a similar expression as in equation (3.19)

expect $R_i(t, \tau; \underline{x})$ is replaced by $R_{ij}(t, \tau; \underline{x})$

$$\begin{aligned} \text{cov}\{W, W_j\} = & \sum_{i=1}^K \left[\frac{1}{N_0^4} \iiint R_i^*(t, \tau_1; \underline{x}_t) R_i(t, \tau; \underline{x}) R_i^*(t_1, \tau_1; \underline{x}_t) R_{ij}(t_1, \tau_1; \underline{x}_t) dt d\tau dt_1 d\tau_1 \right. \\ & + \frac{1}{N_0^3} \iiint R_i^*(t, \tau_1; \underline{x}_t) R_i(t, \tau; \underline{x}) R_{ij}(\tau, \tau_1; \underline{x}) dt d\tau d\tau_1 \\ & + \frac{1}{N_0^3} \iiint R_i^*(t_1, \tau_1; \underline{x}_t) R_{ij}(t_1, t; \underline{x}) R_i(t, \tau; \underline{x}) dt d\tau dt_1 \\ & \left. + \frac{1}{N_0^2} \iint R_i^*(t, \tau; \underline{x}) R_{ij}(t, \tau; \underline{x}) dt d\tau \right] \quad (3.20) \end{aligned}$$

Substituting in Equations (3.13) and (3.14), we obtain Figures 3.2, 3.3, 3.4. Without loss of generality we will assume that $x_{2t}=0$. Figure 3.2 illustrates the behavior of the expected value of the error signal as a function of $\delta=(x_1-x_{1t})/L$ for two values of SNR and channel spread of 10. Observe that there is a range of δ for which the expected value of the error signal is almost linearly related to δ . This region may be selected as a normal operating region for the tracking loop.

Figure 3.3 demonstrate the behavior of the expected value of the error signal for channels with spread =1.0. Figure 3.4 shows the how the variance of the control signal changes with δ for different values of SNR.

3.4 CONCLUSIONS

A closed loop structure for synchronization over fading dispersive channels, with Gaussian shaped scattering function, is developed. The effects of various system parameters (such as signal-to-noise ratio, the spread of the channel and the duration of the envelope of the transmitted waveform) on the shape of the S-curve are demonstrated. Dependency of the variance of

the tracking error on the SNR and the spread of the channel is investigated.

It is known that in AWGN environment the optimum tracking discriminator for arbitrary wideband signal is a multiplier which forms the product of the received signal plus noise and the first derivative, with respect to the parameter under consideration, of the receiver generated replica of the transmitted signal. This discriminator is optimum in that its output is the maximum likelihood estimate of the that parameter difference between the two wideband signals. In synchronization over fading dispersive channel it is shown that the optimum tracking synchronizer is a multiplier which forms the product of the received signal and a filtered version of it, the filter impulse response is the first derivative with respect to time (or frequency) of the autocorrelation function of the signal part of the received waveform.

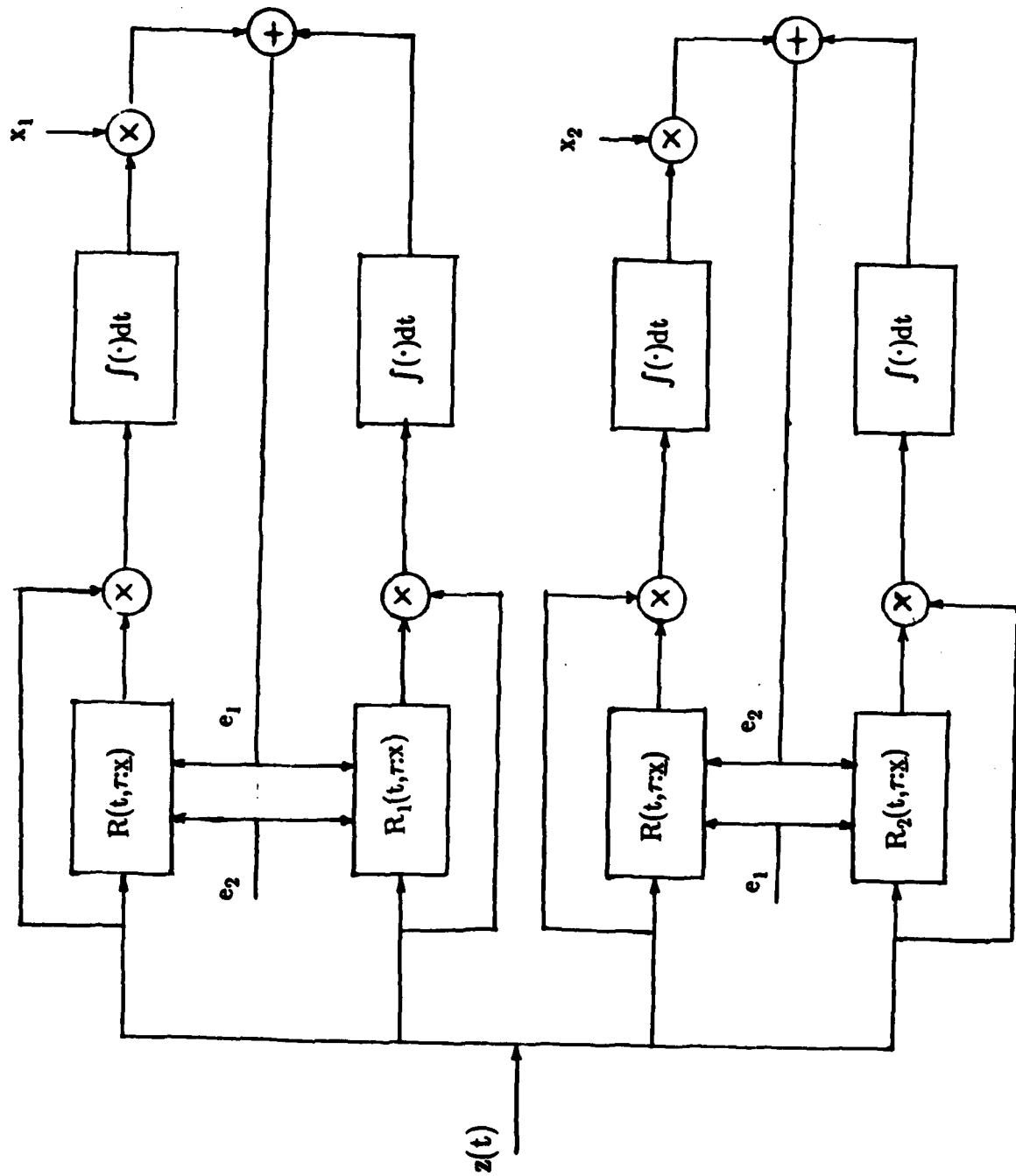


Figure 3.1 Closed Loop Synchronizer Structure

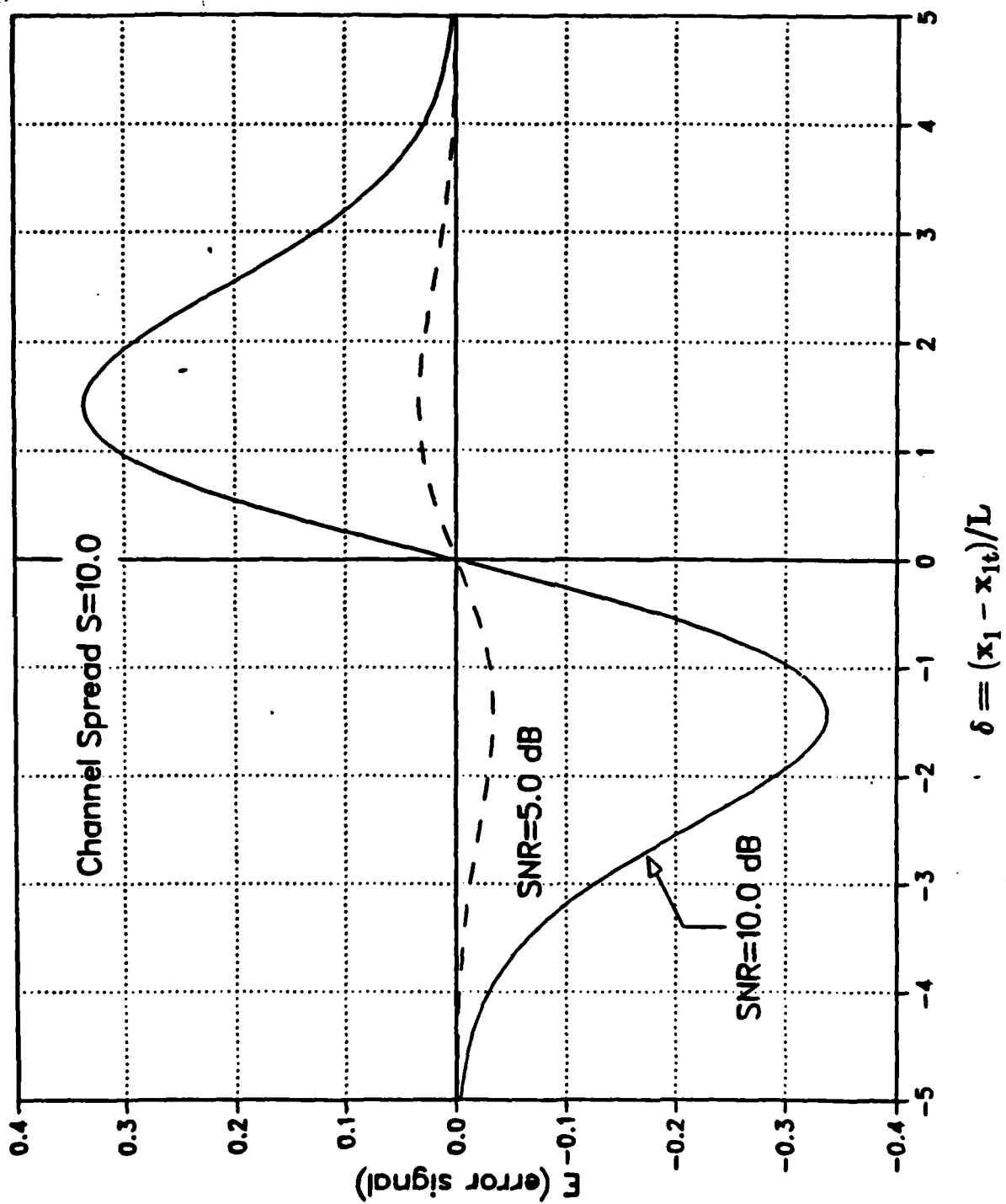


Figure 3.2 Expected Value of the Error Signal

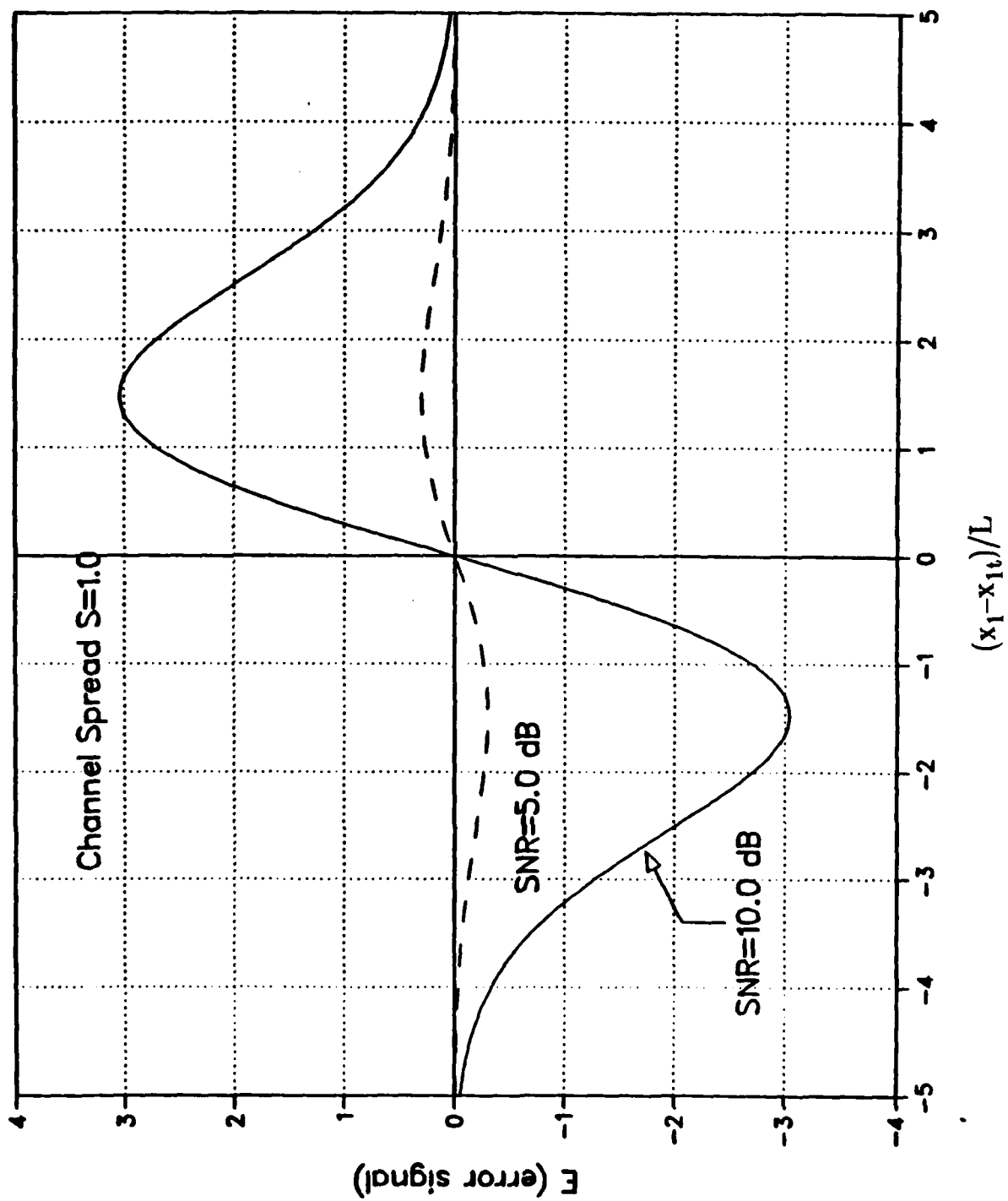


Figure 3.3 Expected Value of the Error Signal

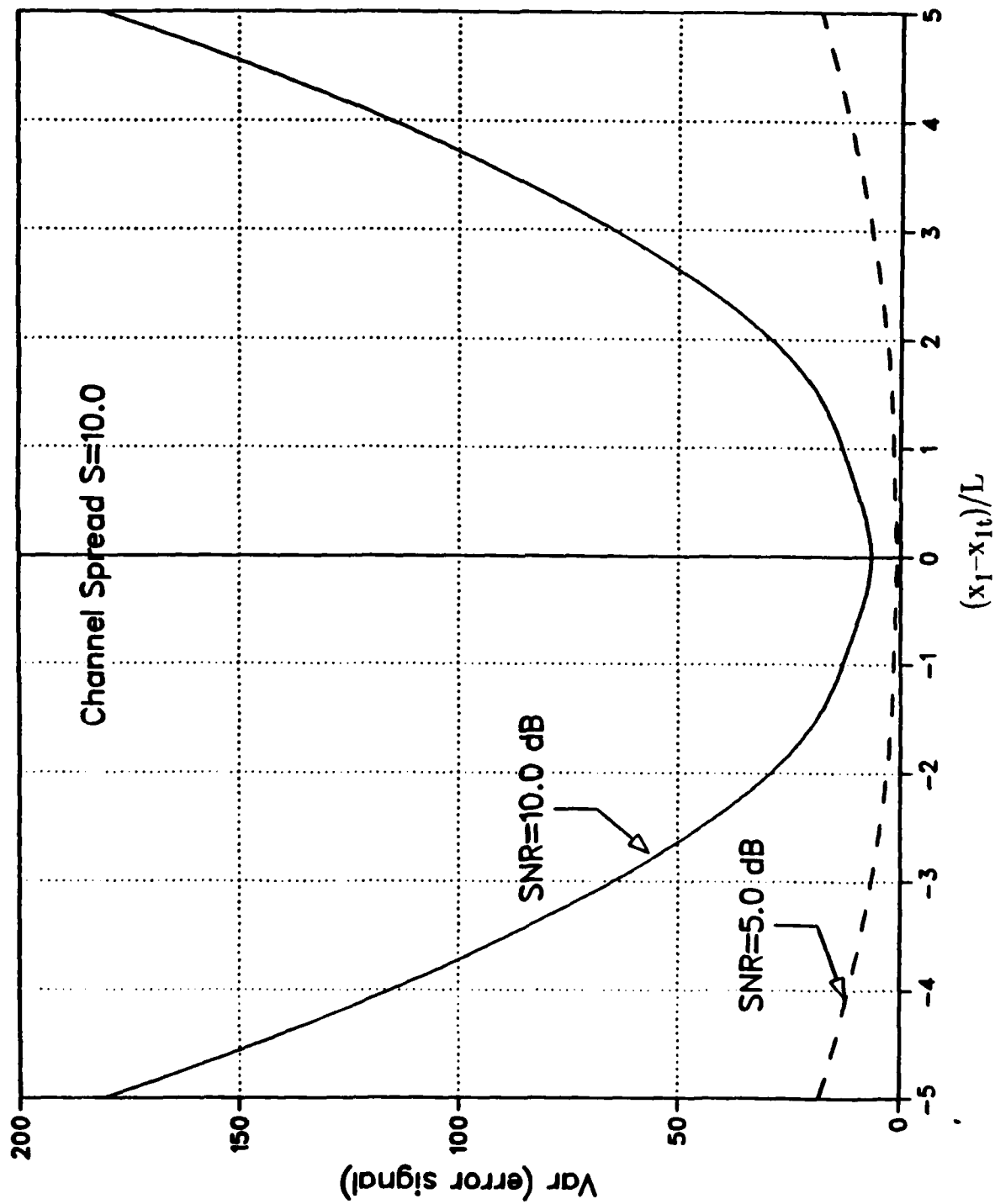


Figure 3.4 Variance of the Error Signal

CHAPTER IV

SYNCHRONIZATION OVER FADING DISPERSIVE CHANNELS USING STOCHASTIC APPROXIMATION METHODS

In this chapter we consider the problem of synchronization over fading dispersive channels. Using stochastic approximations methods; a recursive estimation procedure is developed to estimate the two parameters needed by the synchronizer. The result is a closed loop in which the component of the error signal is proportional to the expected value of the derivative of the likelihood function with respect to the appropriate parameter. This method possesses the simple computational structure of the stochastic approximation methods, and under certain regularity conditions, it can be shown that the variance of the error in the estimates approaches the Cramér-Rao bound. This chapter is organized as follows: In the next section an introduction to the problem is presented, in section 4.2 a brief discussion to the method of stochastic approximation is given. In section 4.3 the method of stochastic approximation is applied to estimate recursively the parameters under consideration. Lastly an example is given in section 4.4 to illustrate that the suggested procedure is asymptotically efficient and derive the conditions under which asymptotic efficiency is

4.1 INTRODUCTION

In this chapter the channel model used is the same one used in section 1.2, and the main objective is to develop a synchronizer which updates itself recursively. In [15] Sakrison described an efficient recursive estimation procedure to estimate some target parameters from repeated observations using

stochastic approximation methods. Under certain conditions these methods provide many computational advantages, and it can be shown that the error in the resulting sequence of the estimates approaches the Cramér-Rao bound.

In the following section a brief coverage of the method of stochastic approximation is given. This method is then applied to the problem of estimating the vector \underline{x} described in section 2.2.

4.2 THE METHOD OF STOCHASTIC APPROXIMATION

Stochastic approximation methods are applicable to any problem that can be formulated as some form of regression problem in which repeated observations are made. To be more specific, let the length of the processing subintervals be denoted by T . Denote the data observed on the k th subinterval by z_k . If our observations start at $t=0$ and the observed quantity is a continuous-time random process, $z(t)$, then z_k represents the sample of the process of T sec duration, $z(t)$, $(k-1)T \leq t < kT$. The objective is to find the value of an unknown parameter \underline{x} which solves the vector valued equation

$$m(\underline{x}) \triangleq E\left\{f(z_k, \underline{x})\right\} = m_o \quad (4.1)$$

Robbins and Monro [16] originally studied an iterative procedure of solving this problem from repeated independent observations in which f , m , and z are scalar valued. A multidimensional version of this procedure is described in Sakrison [17] where it is shown that the sequence of estimates $\{\underline{x}_k\}$ chosen according to the recursion relation

$$\underline{x}_{k+1} = \underline{x}_k + c_k \left[f(z_k, \underline{x}_k) - m_o \right] \quad (4.2)$$

converges to the solution vector \underline{x}_t in the mean square sense,

$$\lim_{k \rightarrow \infty} E \left\{ \|\underline{x}_k - \underline{x}_t\|^2 \right\} = \lim_{k \rightarrow \infty} E \left\{ \sum_{i=1}^2 (x_{k,i} - x_{t,i})^2 \right\} = 0 \quad (4.3)$$

provided that the following assumptions are satisfied.

1. There exist constants a_0 and a_1 , $0 < a_0 \leq a_1 < \infty$, such that

$$a_0 \|\underline{x} - \underline{x}_t\|^2 \leq (\underline{x} - \underline{x}_t)^T (\underline{m}(\underline{x}) - \underline{m}_0) \leq a_1 \|\underline{x} - \underline{x}_t\|^2. \quad (4.4)$$

where \underline{x}^T represent the transpose of the vector \underline{x} .

2. The random entities $z_i, i=1,2,\dots$ are identically distributed and statistically independent.

3. For all values of \underline{x}

$$\text{var} \left\{ \|f(z_k, \underline{x})\|^2 \right\} < \infty. \quad (4.5)$$

4. The sequence of constants $\{c_k\}$, $k=1,2,\dots$ are positive monotone decreasing, and satisfy

$$\sum_{k=1}^{\infty} c_k = \infty, \quad \sum_{k=1}^{\infty} c_k^2 < \infty. \quad (4.6)$$

The usage of the stochastic approximation methods yields the following advantages:

1. Only a small interval of data needs to be processed at a time.
2. Only simple computations are required, even when the functional dependence of the regression function on the parameters of interest is nonlinear.
3. The method may be employed in the absence of the detailed knowledge of the process statistics and in the absence of the detailed knowledge of the relationship between the desired parameters and the observed data.

If sufficient a priori knowledge concerning the statistics of $z(t)$ and the functional relationship between the parameters and the observed data is available, the third advantage can be replaced by the following desirable property: the methods can be made to be asymptotically efficient.

We can apply these results to the synchronization problem under considerations mainly to find an asymptotically efficient Robbins-Monro procedure for estimating \underline{x} from the observations, z_k , $k=1,2,\dots$. The motivation for this goal is clear; such a method would be computationally simple and yet would have performance which, for a large number of observations, k , could not be surpassed by any other method, no matter how computationally complex.

To see how we might approach this objective, define

$$\underline{\psi}_k(\underline{x}) = \text{grad} \left\{ W_k(\underline{x}) \right\} \quad (4.7)$$

where $W_k(\underline{x})$ is $W(\underline{x})$ defined in (2.3) based on $z(t)$, $(k-1)T \leq t \leq kT$. The gradient in (4.7) is with respect to the partial derivatives $\frac{\partial}{\partial x_j}$, $j=1,2$. We notice that $\underline{\psi}_k(\underline{x})$ is a random vector, even in the absence of the AWGN, and a useful characterization is the its expected value. Define the vector-valued function $\underline{m}(\underline{x})$ as

$$\underline{m}_k(\underline{x}) = \mathbf{E} \left\{ \underline{\psi}_k(\underline{x}) \right\}. \quad (4.8)$$

We also notice that

$$\underline{m}_k(\underline{x}) \begin{cases} = 0 & \underline{x} = \underline{x}_t \\ \neq 0 & \text{in general} \end{cases} \quad (4.9)$$

Thus if Eq. (4.9) has only the unique solution $\underline{x} = \underline{x}_t$, we can hope to carry out Robbins-Monro method for estimating \underline{x}_t by using the $\underline{\psi}_k(\underline{x})$ of Eq. (4.7) in the procedure.

The most rapid convergence is obtained when the weighting constants c_k are chosen to be of the form c/n . It can be shown that [17] if $c = G^{-1}(\underline{x}_t)$, where the ij th entry of the matrix $G(\underline{x}_t)$ is given by

$$\begin{aligned} g_{ij}(\underline{x}_t) &= E \left\{ \frac{\partial^2 W(\underline{x}_t)}{\partial x_{it} \partial x_{jt}} \right\} \\ &= E \left\{ \frac{\partial \psi_{k,j}(\underline{x}_t)}{\partial x_{it}} \right\}, \end{aligned} \quad (4.10)$$

then the Robbins-Monro method is asymptotically efficient. The only problem is how to pick $c = G^{-1}(\underline{x}_t)$? since \underline{x}_t is unknown. One solution is to make the procedure adaptive by substituting an estimate for $G(\underline{x}_t)$. The "adaptive" Robbins-Monro procedure takes the form

$$\underline{x}_{k+1} = \underline{x}_k + (1/k) G^{-1}(\underline{x}) \psi_k(\underline{x}). \quad (4.11)$$

Sakrison [18] considered the convergence of this method. Under certain assumptions, Sakrison has shown that $C_{\underline{x}_k}$, the covariance matrix of the error $(\underline{x}_k - \underline{x}_t)$, satisfies

$$(\underline{b}, C_{\underline{x}_k} \underline{b}) \leq (1/k)(\underline{b}, G \underline{b}) + d/k^{1+\gamma}, \quad d < \infty, \gamma > 0 \quad (4.12)$$

where $(\underline{x}, \underline{y})$ is the inner product of the in the M -dimensional Euclidean space, and \underline{b} is an arbitrary vector. That is, the sequence of estimates \underline{x}_k generated by Eq. (4.11) is asymptotically efficient. The interested reader is referred to [18] for the proof of the last statement.

4.3 NUMERICAL RESULTS

Combining Eqs. (2.3) and (4.10), yields

$$m_{k,j} = \frac{1}{N_o^2} \int_0^T \int_0^T \left[R^*(t, \tau; \underline{x}_t) - N_o \delta(t - \tau) \right] R^j(t, \tau; \underline{x}) dt d\tau \quad j=1,2; \quad (4.13)$$

where superscript denote partial differentiation with respect to the corresponding component of \mathbf{x} . In order to proceed further we need to specify the shape of the scattering function, $\sigma(r, f)$. Let

$$\sigma(r, f; \mathbf{x}) = \frac{E_r}{2\pi LB} \exp \left[-(r-x_1)^2/2L^2 - (f-x_2)^2/2B^2 \right], \quad (4.14)$$

also assume that the complex envelope of the transmitted signal be

$$u(t) = \left(\frac{2\alpha}{\pi} \right)^{1/4} \exp \left[-\alpha t^2 \right] \quad (4.15)$$

Substituting in Eq. (1.3) yields

$$R(t, \tau; \mathbf{x}) = \frac{E_r \sqrt{\frac{2\alpha}{\pi}}}{\sqrt{\beta}} \exp \left[-\frac{\alpha}{\beta} ((t-x_1)^2 + (\tau-x_2)^2 + 2\alpha L^2 (t-\tau)^2) \right. \\ \left. + j2\pi(\tau-t)x_2 - 2\pi_2 B_2 (\tau-t)_2 \right] \quad (4.16)$$

where

$$\beta = 1 + 4\alpha L^2. \quad (4.17)$$

It is known that [4] the optimum value for α is

$$\alpha = \pi \frac{B}{L} \quad (4.18)$$

The elements of the matrix $G(\mathbf{x})$ are

$$g_{11} = 2 \gamma (2\gamma \Delta x_1^2 - 1) Q \quad (4.19-a)$$

$$g_{12} = g_{21} = 4 \gamma^2 \Delta x_1 \Delta x_2 Q \quad (4.19-b)$$

$$g_{22} = 2 \gamma (2\gamma \Delta x_2^2 - 1) Q \quad (4.19-c)$$

where

$$\Delta x_1 = \frac{x_1 - x_{1t}}{L}, \quad \Delta x_2 = \frac{x_2 - x_{2t}}{B}, \quad \gamma = \frac{\pi S}{\beta} \quad (4.20)$$

and

$$Q = \frac{\text{SNR}^2}{\beta} \exp \left[-\gamma (\Delta x_1^2 + \Delta x_2^2) \right]. \quad (4.21)$$

The vector $\underline{m}(\underline{x})$ is

$$\underline{m}(\underline{x}) = -2 Q \gamma \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} \quad (4.21)$$

From (4.21) and (4.19), the quadratic form $q = -(\underline{x} - \underline{x}_t)^T G(\underline{x}) \underline{m}(\underline{x})$ is

$$q = 8\gamma Q^2 \left\{ L \Delta x_1^2 (\gamma^2 \Delta x_1^2 + \gamma^2 \Delta x_2^2 - \gamma) + B \Delta x_2^2 (\gamma^2 \Delta x_2^2 + \gamma^2 \Delta x_1^2 - \gamma) \right\} \quad (4.22)$$

4.4 CONCLUSIONS

The ML solution is the one for which $\frac{\partial W}{\partial x_i} = 0$. In the case of fading dispersive channels, the decision variable, even in the absence of the AWGN, is a random variable, a useful characterization is the expectation of W . One way to do approximate this solution is to update the estimate by a quantity proportional to the error. We wrote the error function as

$$e(\underline{x}) = \mathbf{E} \left\{ e(\underline{x}) \right\} + (e(\underline{x}) - \mathbf{E} \left\{ e(\underline{x}) \right\})$$

The first term is used to control the new estimate and the second term is treated as an internisic noise. Eq. (4.11) simply indicates that we update the estimate of any component of \underline{x} by using a quantity proportional to the error. The proportionality factor is chosen to speed up the convergence process. The resulting estimate $\hat{\underline{x}}$ is asymptotically efficient as long as $\underline{x} - \underline{x}_t$ lies in a given region. which we called the pull-in region.

REFERENCES

- [1] P.A. Bello, "Characteristics of Randomly Time-Variant Linear Channels," *IEEE Transactions on Communication Systems*, Vol. CS-11, pp.360-393, Dec. 1963.
- [2] L.A. Zadah, "Frequency Analysis of Variable Network," *Proceedings of IRE*, Vol. 38, pp. 291-299, March 1950.
- [3] R.S. Kennedy, *Fading Dispersive Communication Channels*, John Wiley, 1968.
- [4] S.S. Soliman, "Synchronization over Fading Dispersive Channels," Ph.D Thesis, University of Southern California, Los Angeles, California, 1983.
- [5] H.L. Van Trees, *Detection, Estimation and Modulation Theory*, Part 3, New York: John Wiley, 1971.
- [6] H.L. Van Trees *Detection, Estimation, and Modulation Theory*, Part I, pp. 250-253, Wiley, New York, 1968.
- [7] W.C. Lindsey and M.K. Simon, *Telecommunication Systems Engineering*, Prentice-Hall, Englewood Cliffs, N.J. 1973.
- [8] J.J. Stiffler, *Theory of Synchronous Communications*, Prentice-Hall, Englewood Cliffs, N.J., 1971.
- [9] A.J. Viterbi, *Principles of Coherent Communication*, McGraw-Hill, New York, 1966
- [10] W.C. Lindsey and M.K. Simon, *Synchronization Systems in Communication and Control*, Prentice-Hall, Englewood Cliffs, N.J., 1971.
- [11] S.S. Soliman and R.A. Scholtz, "Autocorrelation Ambiguity Functions," submitted to *IEEE Trans. Inform. Theory*.

- [12] J.K. Holmes and C.C. Chen, "Acquisition Time Performance of PN Spread Spectrum Systems," *IEEE Trans. Commun.*, Vol. COM-25, pp. 778-784, August 1977.
- [13] S.S. Soliman, "Serial Search Synchronization Over Fading Dispersive Channels," *IEEE MONTECH '86, Conference on Antennas and Communications, Montreal, Canada, Sept. 1986*.
- [14] E.N. Fowle, D.R. Carey, R.E. Vander and R.C. Yost "A Pulse Compression System Employing a Linear FM Gaussian Signal," *Proc. IEEE*, pp.304-312, Feb. 1963.
- [15] D.J. Sakrison, "Efficient Recursive Estimation of the Parameters of a Radar or Radio Astronomy Target," *IEEE Trans. Inform. Theory*, Vol. 12, pp. 35-41, January 1966.
- [16] H. Robbins and S. Monro, "A Stochastic Approximation Method," *Ann. Math. Statist.*, Vol. 22, pp. 400-407, September 1951.
- [17] D.J. Sakrison, "Stochastic Approximation, A Recursive Method for Solving Regression Problems," in *Advances in Communication Systems*, A.V. Balakrishnan, Ed. , pp. 51-105, Academic Press, New York, 1966.
- [18] D.J. Sakrison, "Efficient Recursive Estimation; Application to Estimating the Parameters of a Covariance Function," *Int. Journal Eng. Sci.* Vol. 3, pp. 461-483, 1965.

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